Taking Some Burden off an Explicit CTL Model Checker

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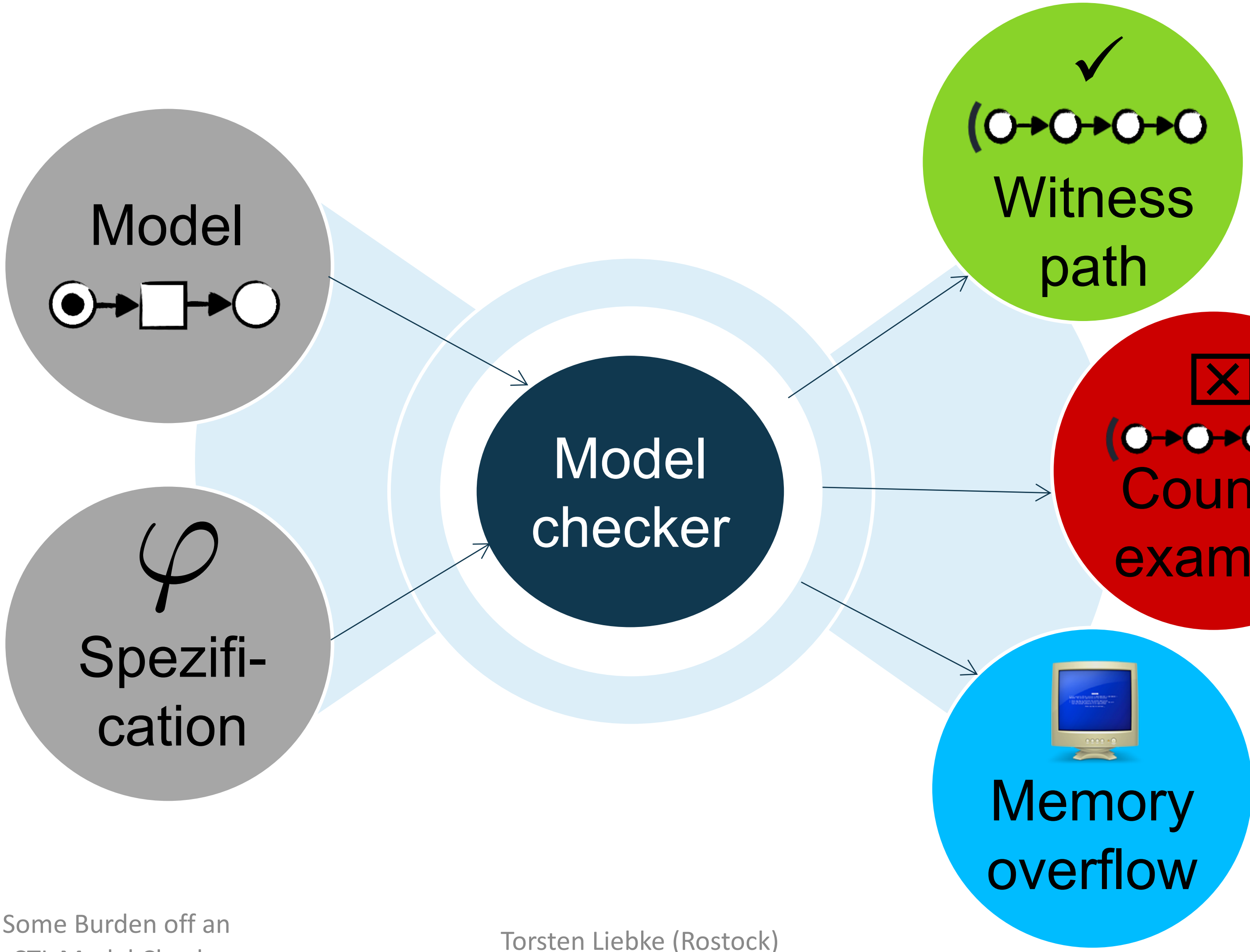
Model checking



Computational Tree logic (CTL)

$$
\begin{aligned}
\phi ::= {} & \mathtt{T} \mid \mathtt{F} \mid \mathtt{FIREABLE} \mid \mathtt{DEADLOCK} \mid \rho \\
& \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \longrightarrow \phi \\
& \mid \mathrm{AX}\ \phi \mid \mathrm{EX}\ \phi \\
& \mid \mathrm{AF}\ \phi \mid \mathrm{EF}\ \phi \\
& \mid \mathrm{AG}\ \phi \mid \mathrm{EG}\ \phi \\
& \mid \mathrm{A}\ (\phi\ \mathrm{U}\ \phi) \mid \mathrm{E}\ (\phi\ \mathrm{U}\ \phi)
\end{aligned}
$$

Path quantifier

A: inevitably (along all paths)

E: possibly (there exists a path)

Temporal operators

G: globally (always)

F: in future (eventually)

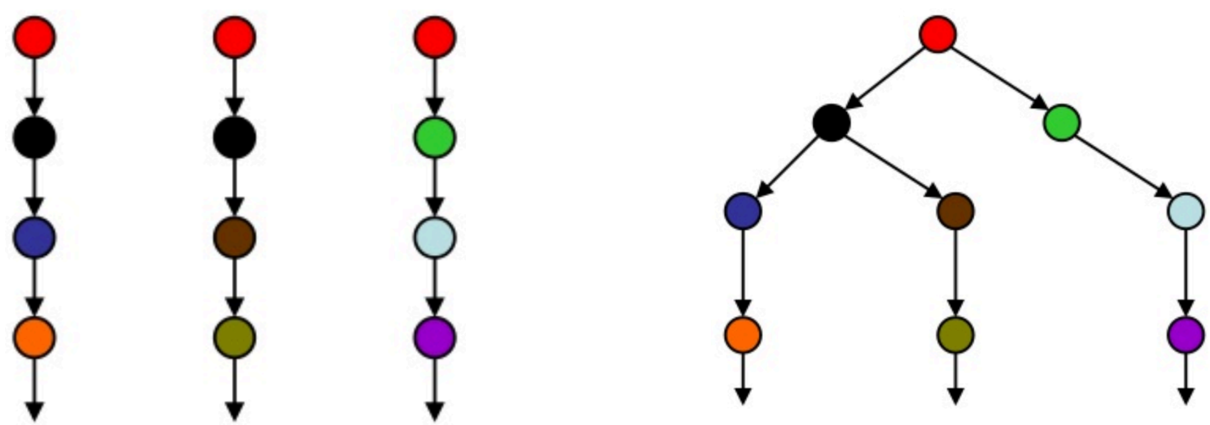
X: neXt state

U: until

Linear Time Logic (LTL)

$$\phi ::= T \mid F \mid \mathrm{FIREABLE} \mid \mathrm{DEADLOCK} \mid \rho$$
$$\mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \longrightarrow \phi$$
$$\mid X\ \phi \mid F\ \phi \mid G\ \phi \mid (\phi\ U\ \phi)$$

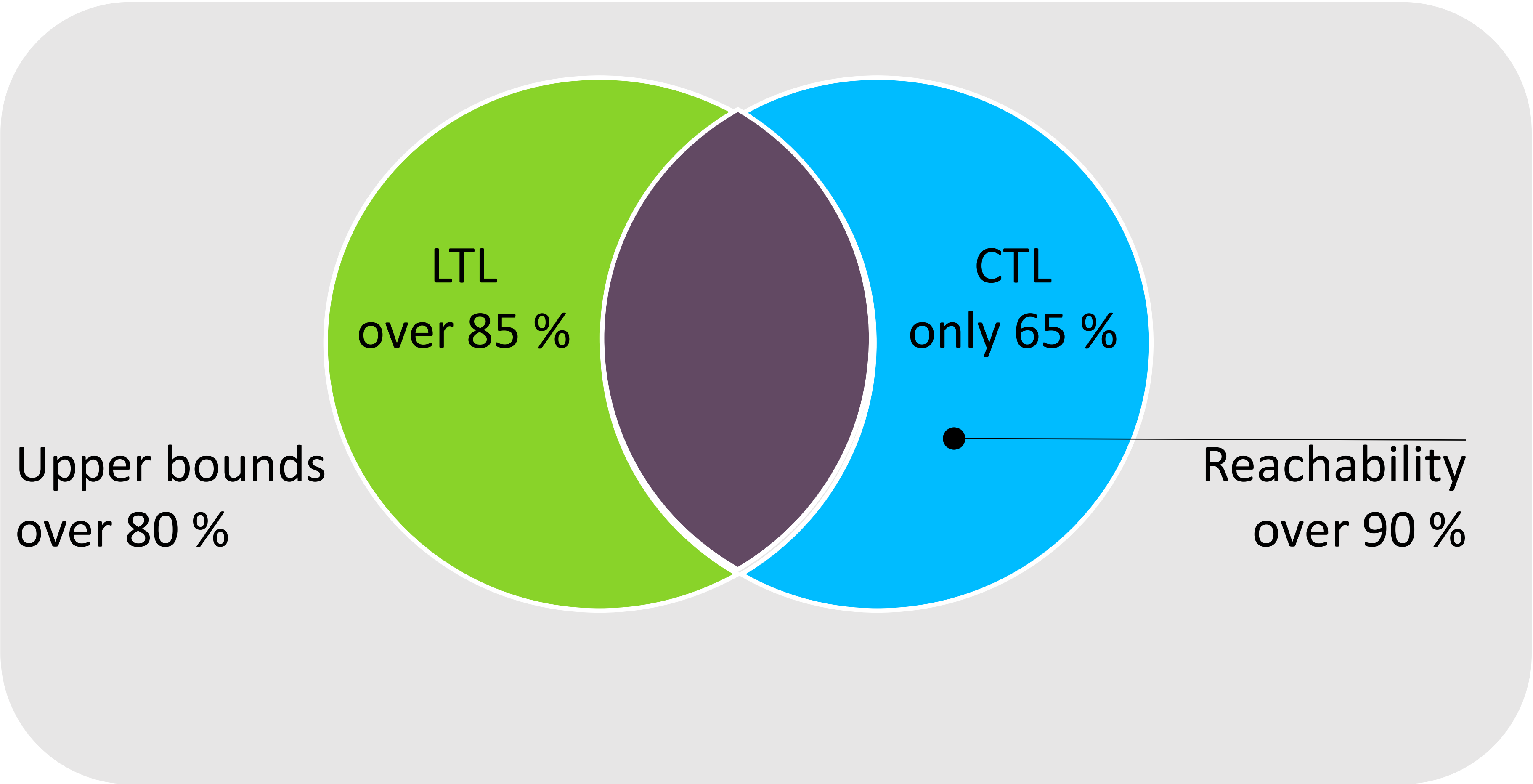
Similar to CTL but path quantifiers are not used.



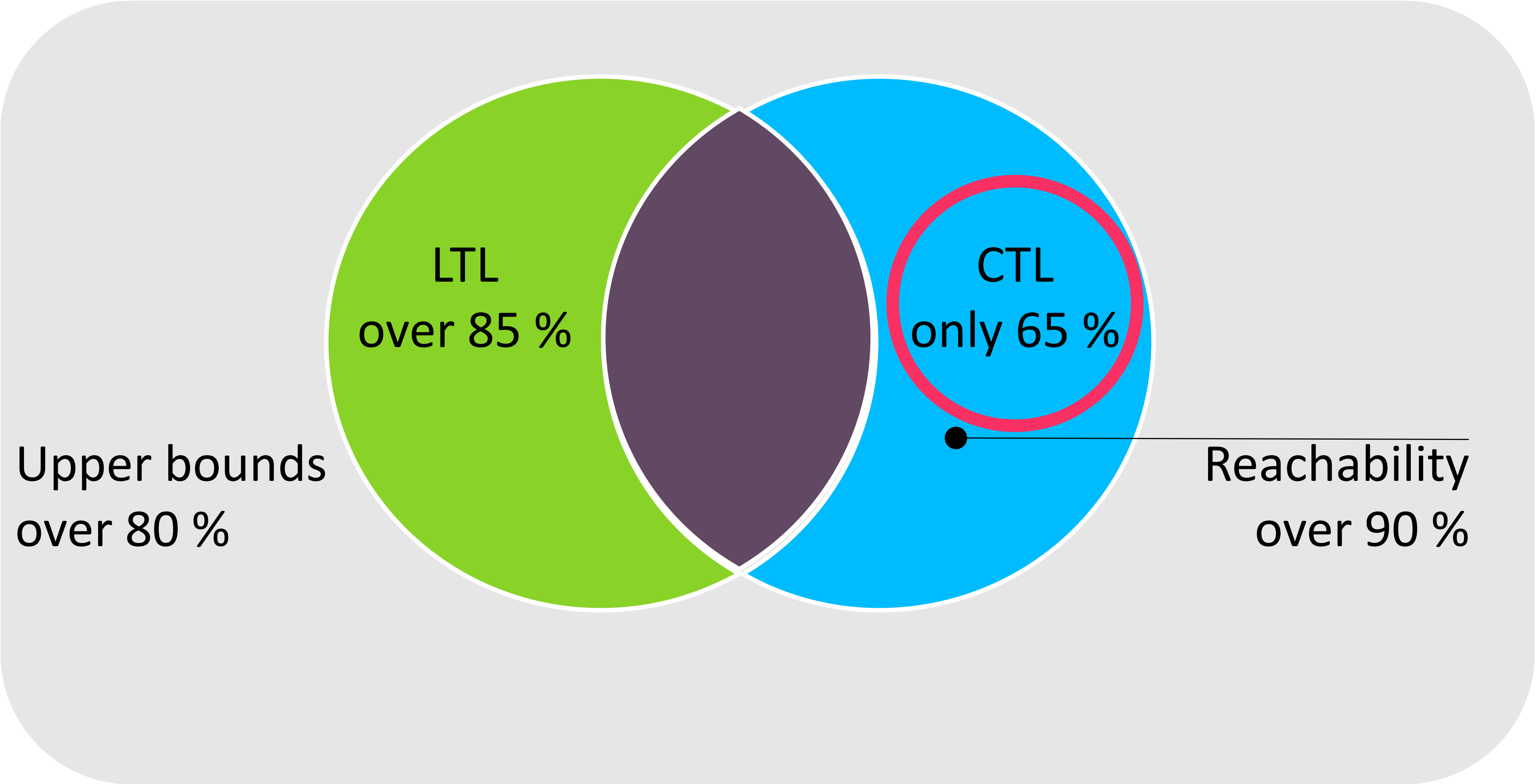
LTL: linear time

CTL: branching time

LoLA’s performance at the MCC



CTL performs worse than the rest: LoLA's performance at the MCC



Simple and frequently occurring formulas

Many CTL queries have a rather simple structure – only few temporal operators

- In the MCC this could be an artefact of the randomised formula generating mechanism
- Share same experience with LoLA users

Goal

Find for each formula an optimized algorithm

Systematic approach

- Most approaches we're using are well known
- Combining them in a systematic way, to push the limits further

Building a uniformly picture

$\Rightarrow$ Especially for stubborn sets

$\Rightarrow$ Pre-processing

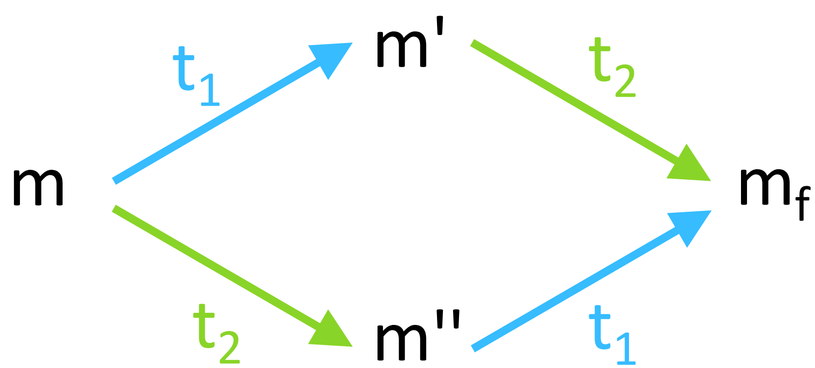
Partial order reduction: The stubborn set method

Given: Petri net $N = [P,T,F,W,m_0]$ and property ϕ

Goal: produce subgraph of the reachability graph

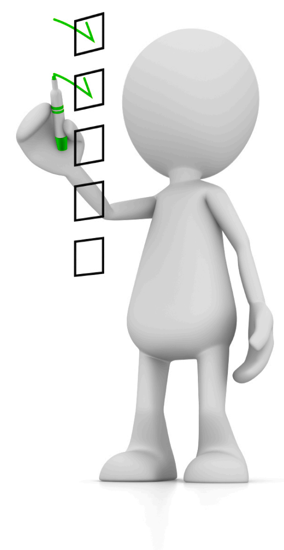
Condition: evaluation of ϕ yields same result

In any given marking, only a subset of the enabled transitions is explored = stubborn(m) $\subseteq T$



Principles

- There exists a list of principles to build the subgraph
- Based on the selected principles, all properties of a certain class are preserved



In the following:

$\boldsymbol{\pi}$‘ = Path in subgraph

$\boldsymbol{\pi}$ = Path in original graph

COM: The commutativity principle

- Transitions may be executed in another order
- Can shift transitions to the front of the path

$$m \overset{w}{\rightsquigarrow} m' \xrightarrow{t} m_f$$

$$m \xrightarrow{t} m'' \overset{w}{\rightsquigarrow} m_f$$

KEY: The key transition principle

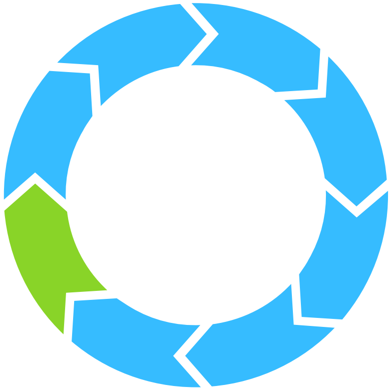
- Transition that stays enabled
- Can push transition to the right

VIS: The visibility principle

- VIS ensures the order of transitions visible for ϕ does not change
- Visible transitions in $\boldsymbol{\pi}$' appear in the same order as in $\boldsymbol{\pi}$, if they appear in $\boldsymbol{\pi}$'

IGN: The non-ignoring principle

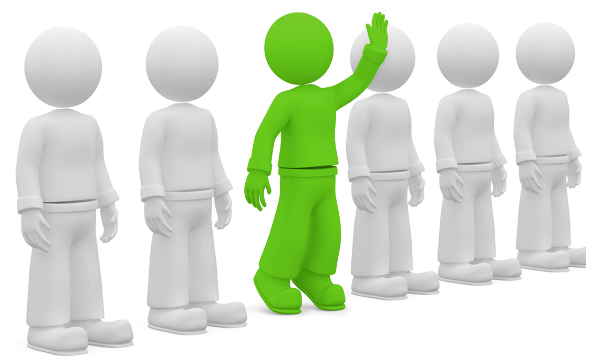
- All transitions are fired at least once in every circle
- Ensures that all transitions of $\boldsymbol{\pi}$ are eventually occurring in $\boldsymbol{\pi}$‘



All transitions are enabled

UPS: The up-set principle

- Stubborn set at m will always contain a transition of $\boldsymbol{\pi}$
- Between current marking and final marking there is a transition from up-set



BRA: The branching principle

- Ensures that visible transitions are not swapped with branches in the state space other than branches that are introduced by concurrency
- Enables reduction only in markings where just one (invisible) enabled transition is sufficient to meet all other principles

Partial order reduction for CTL

- Has severe restrictions
- Either a singleton set of an invisible transition, that satisfies all other criteria ...

Partial order reduction for CTL

- Has severe restrictions
- Either a singleton set of an invisible transition, that satisfies all other criteria ...

- Or we have to fire all enabled transitions
- Necessary to preserve BRA

Good news!

In all reported cases the very limiting BRA principle can be dropped.

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In all reported cases the very limiting BRA principle can be dropped.

In General:

less restrictive conditions (i.e. smaller set of principles to be met)

$\Rightarrow$ potentially smaller stubborn-sets

$\Rightarrow$ better reduction

BRA: enables reduction only in markings where just one (invisible) enabled transition is sufficient to meet all the other principles

AG ϕ, EF ϕ

AG ϕ = invariant, EF ϕ = reachability

- There are already well known stubborn sets
- Reachability: over 90 % - CTL only 65 %

$\Rightarrow$ Use the reachability stubborn sets

Structural analysis can also be applied:

- State equation with the CEGAR approach
- EF DEADLOCK – check for Commoner's theorem

EF ϕ – Exists a path, where finally ϕ holds?

AF ϕ, EG ϕ

AF ϕ ≡ F ϕ in LTL

⇒ Use LTL-X preserving stubborn sets (no BRA)

⇒ LTL: 90 % vs. CTL: 65 %

Drop IGN for visible transitions.

⇒ COM, KEY, VIS

⇒ Smaller stubborn sets

EG ϕ

EG ϕ – Exists a path, where permanently ϕ holds?

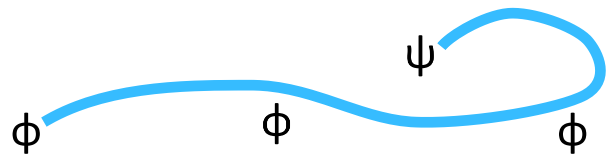
E (ϕ U ψ), A (ϕ R ψ)

E (ϕ U ψ) stubborn sets must preserve two properties:

1. Reachability of ψ
2. Non-violation of ϕ

Combining reachability (EF) and non-violation (EG) stubborn sets:

⇒ COM, UPS(ψ), VIS(ϕ)



E (ϕ U ψ) – Exists a path, where permanently ϕ holds until ψ holds?

EGEF ϕ, AFAG ϕ

No special stubborn sets → CTL-X stubborn sets

But: check for the pair of temporal operators can be folded into a single depth-first search

Two cases:

1. Deadlock: deadlock-state has to satisfy ϕ
2. Loop: from marking m on the loop, marking m’ satisfying ϕ is reachable

EGEF ϕ – Exists a path, where permanently EF ϕ holds ?

EFEG ϕ, AGAF ϕ

Nested depth-first search

Inner search: proceeds only through ϕ-markings and tries to find a cycle or a deadlock

$\Rightarrow$ COM, KEY, VIS(ϕ)

Outer search: proceeds through markings that have already proven not to be part of a ϕ-cycle (or a ϕ-deadlock)

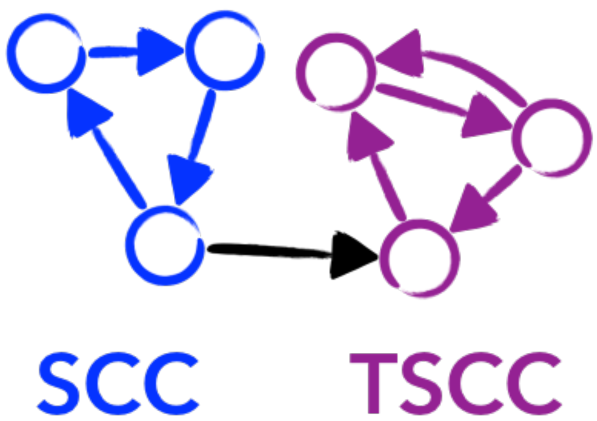
$\Rightarrow$ $m \not\models \phi$: COM, UPS(ϕ)

$\Rightarrow$ $m \models \phi$: COM, UPS(¬ϕ)

EFEG ϕ – Exists a path to a ϕ-loop or ϕ-deadlock?

AGEF ϕ, EFAG ϕ, AGEFAG ϕ, EFAGEF ϕ

- Only TSCC are relevant
- Use existing TSCC preserving stubborn sets



Formulas starting with EX and AX

Check the respective formula without the leading EX operator.

All we need to do is:

- explore all enabled transitions of m_0
- not store m_0

Whenever m_0 is visited during the search, it is treated as fresh marking.

Single-path formulas

- Aim: apply LTL model checking instead of CTL

$\Rightarrow$ BRA principle may be skipped

- Use rewriting system to recognise qualified formulas

Existential single-path formulas:

ϕ and ψ are existential single-path formulas, ω is a state predicate

- ω (base of inductive definition)
- $EG\ \omega$
- $EF\ \phi$
- $E\ (\omega\ U\ \phi)$
- $E\ (\phi\ R\ \omega)$
- $\phi \vee \psi$
- $\phi \wedge \omega$

Universal single-path formula are defined similar.

Boolean combinations

CTL formula is Boolean combination of subformulas

$\Rightarrow$ Check subformulas individual

Advantage:

- Smaller set of visible transition
- Apply stubborn sets to formulas without X-operator
- Some fall into the class considered above

Quick checks

For quite a few formulas we can add sufficient or necessary quick checks

E.g. AGEF ϕ → EF ϕ = nec., AG ϕ = suff.

State equation with CEGAR can be used

⇒ Not much memory used

⇒ Can run in parallel

⇒ Solved 1.24 % in MCC'2018

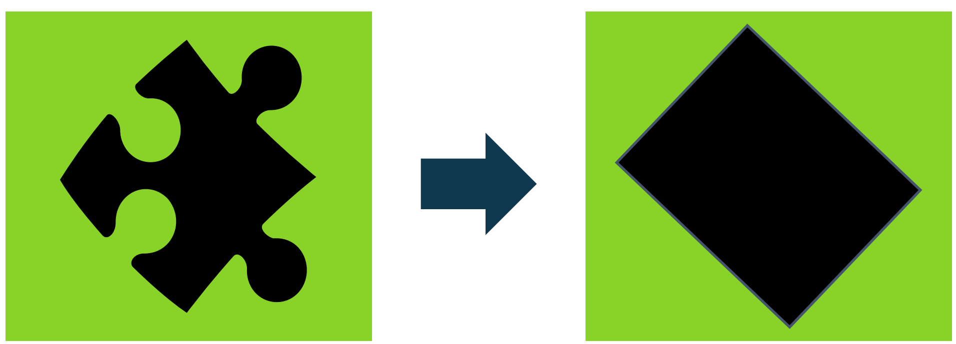
Simplify complex formulas

Tautologies – not all commonly known

- LoLA contains more than 100 rewrite rules based on CTL* tautologies

ILP-techniques using the Petri net state equation can be applied to atomic propositions

- Sometimes proving them invariantly true or false



Distribution in the MCC’2018 (Place/transition nets only)

	# of formulas	In %
All	24544	100,0
Special	13366	54,46
Preprocessing	3704	15,09
CTL	7474	30,45

Statistics

Formula	All	Solved CTL	Solved special	Solved more absolut	Solved more in %	Solved remaining in %
E(F(*)) / A(G(*))	2471	1438	2300	862	34,9	83,4
E(G(*)) / A(F(*))	1767	1625	1670	45	2,5	31,7
E((* R *)) / A((* U *))	168	157	160	3	1,8	27,3
E((* U *)) / A((* R *))	318	187	198	11	3,5	8,4
E(F(E(G(*)))) / A(G(A(F(*))))	515	340	431	91	17,7	52,0
E(G(E(F(*)))) / A(F(A(G(*))))	385	276	277	1	0,3	0,9
E(X(...)) / A(X(...))	602	407	539	132	21,9	67,7
Single-path formulas	421	275	295	20	4,8	13,7
TSCC-based	897	289	349	60	6,7	9,9
Boolean	5822	4250	5239	989	17,0	62,9
All	13366	9244	11458	2214	16,6	**53,7**

Summary

Simplify complex formulas

Use special features of the formula

Necessary / sufficient quick checks

CTL over 80%

Time for discussion!