Parameterized Analysis of Immediate Observation Petri Nets

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joint work with Javier Esparza and Mikhail Raskin

European Research Council

Established by the European Commission

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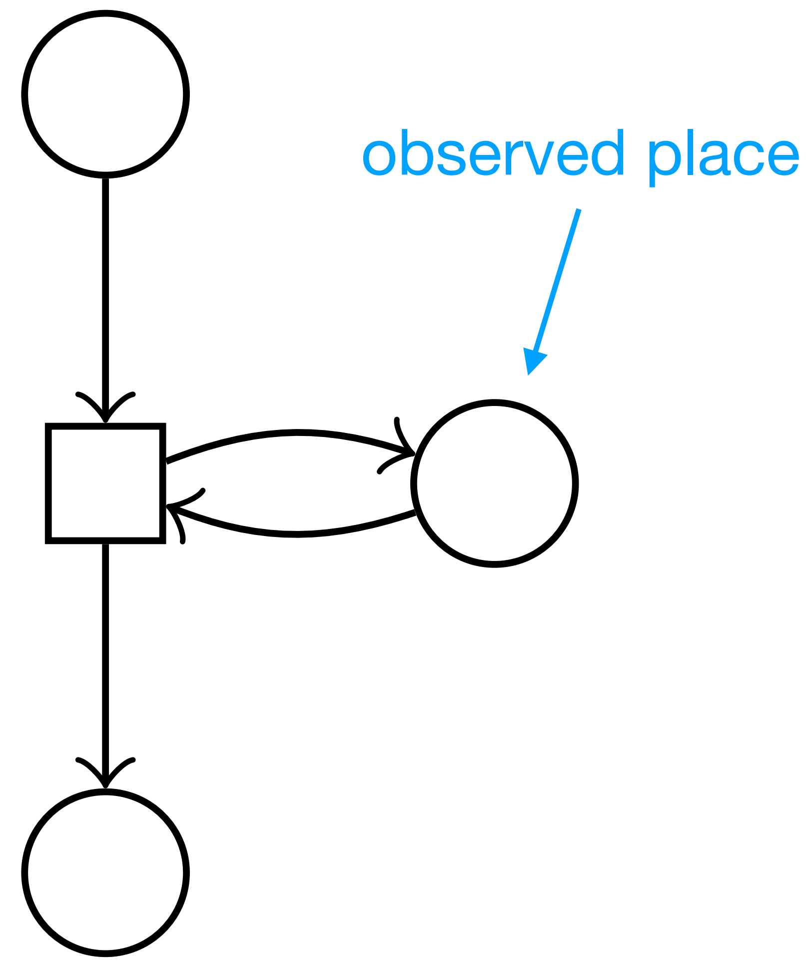
Immediate Observation Petri Nets

All transitions have

- one input place and one output place, and
- one observed place that is both input and output.



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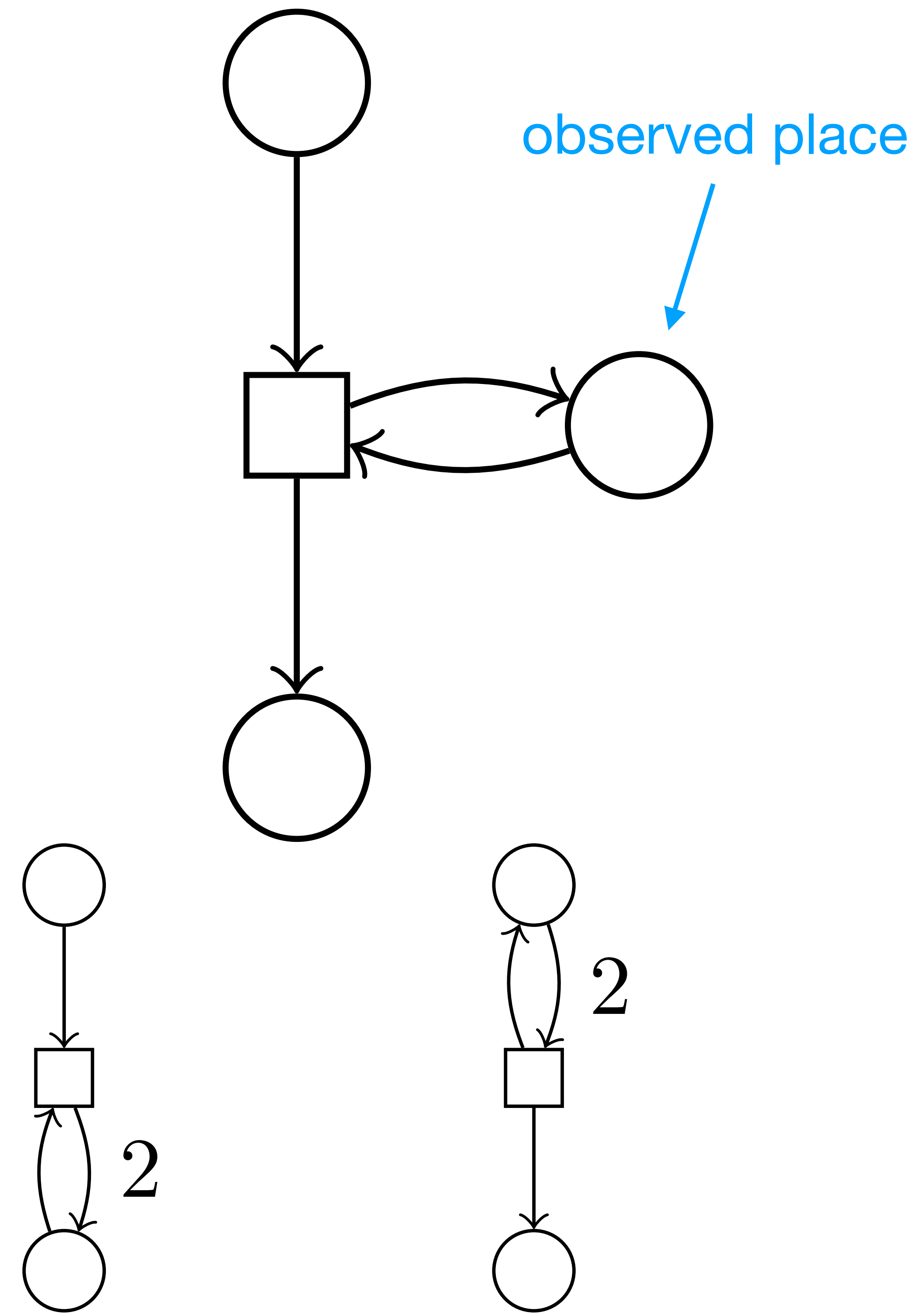

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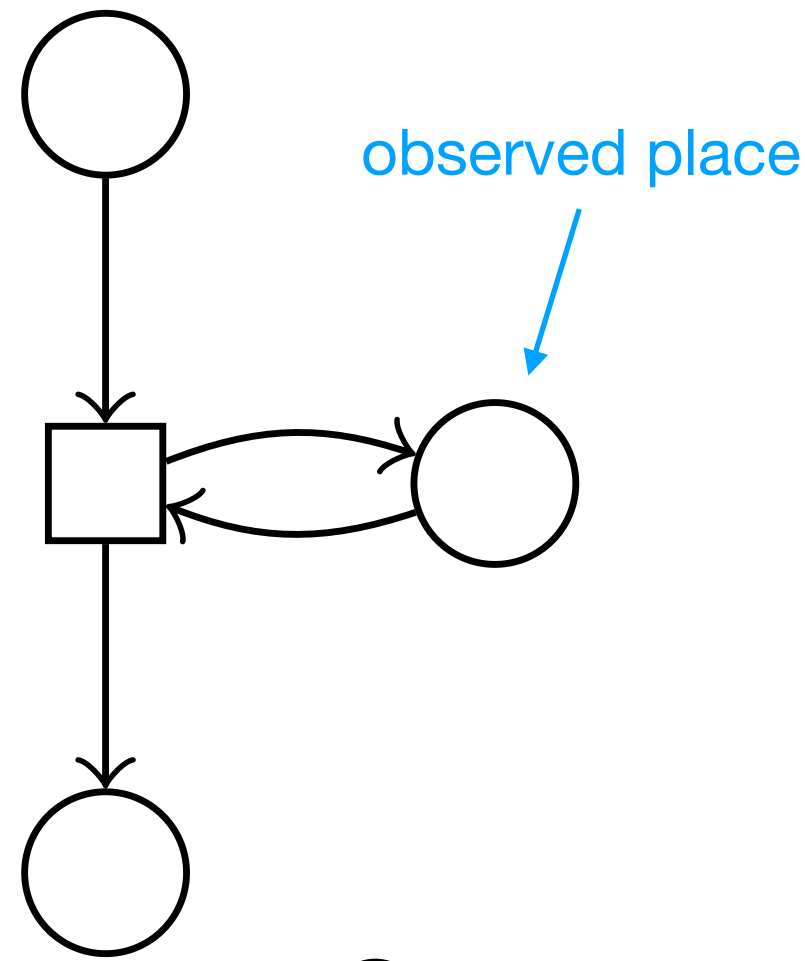
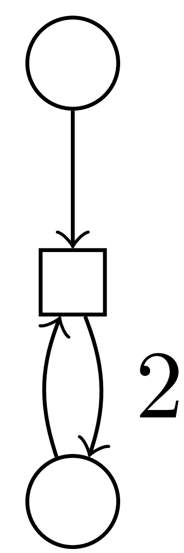
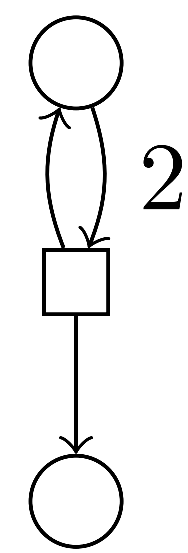


IO nets are **conservative**.





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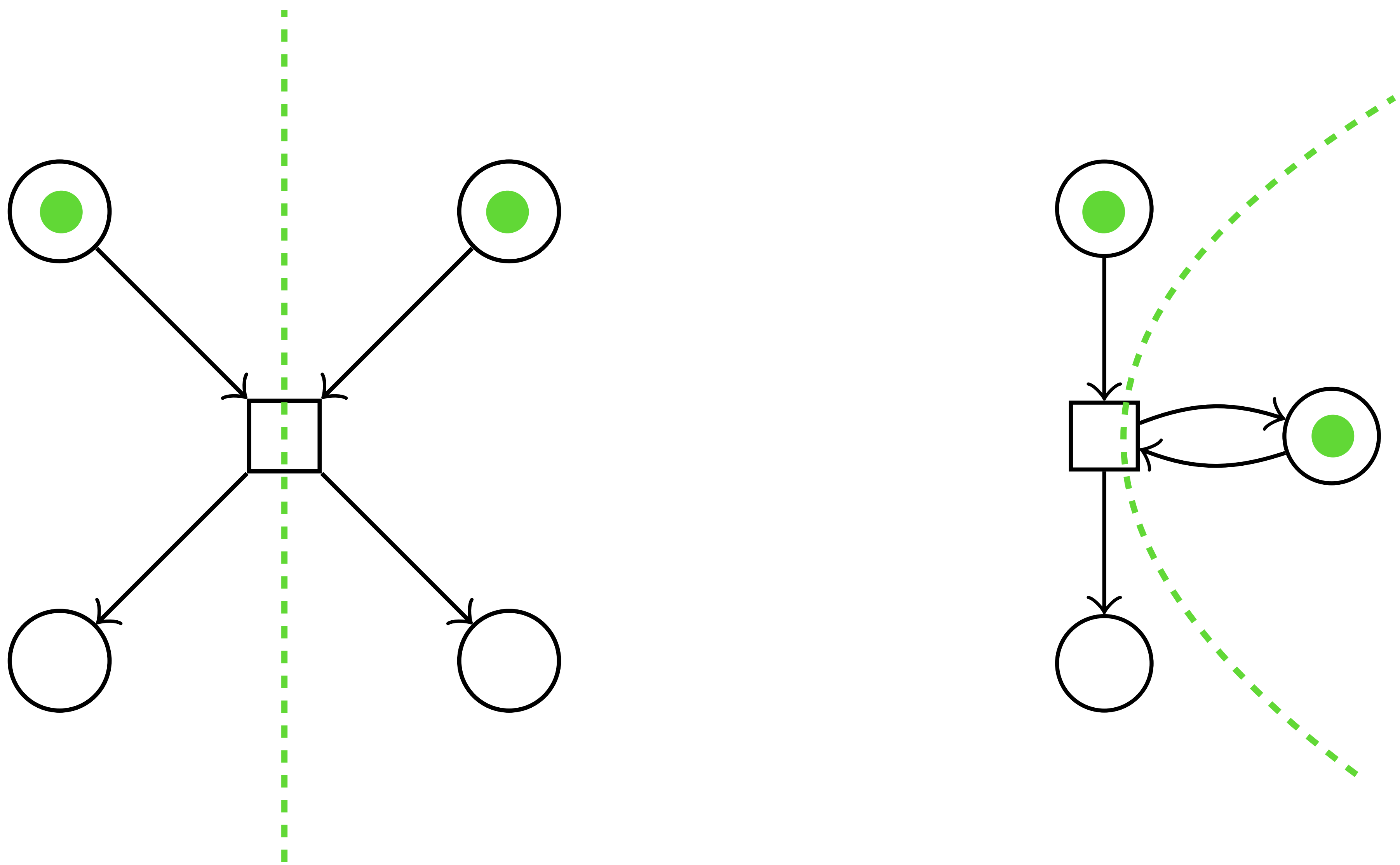

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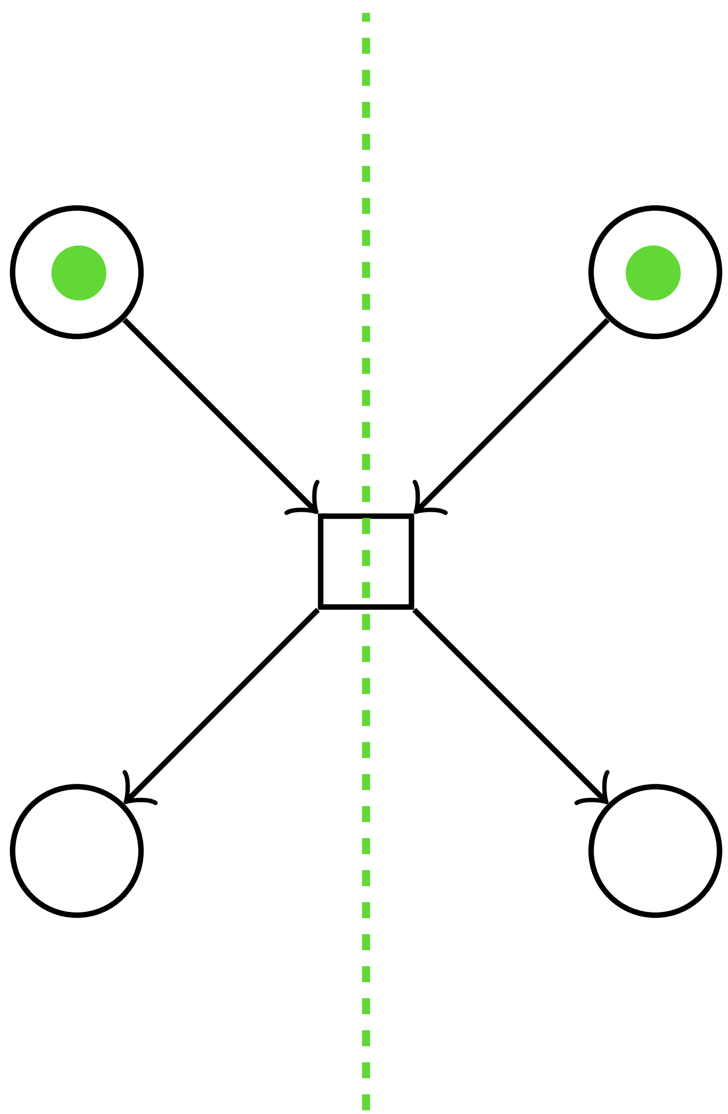
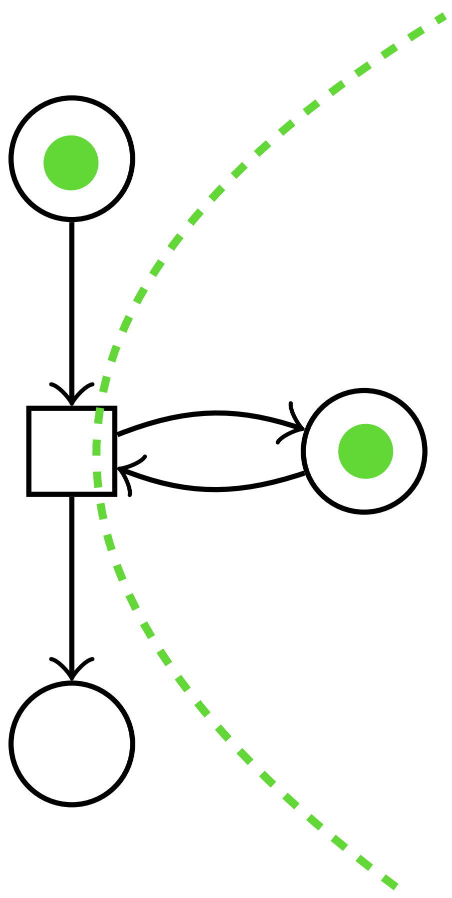


Processes interact and change
their states simultaneously



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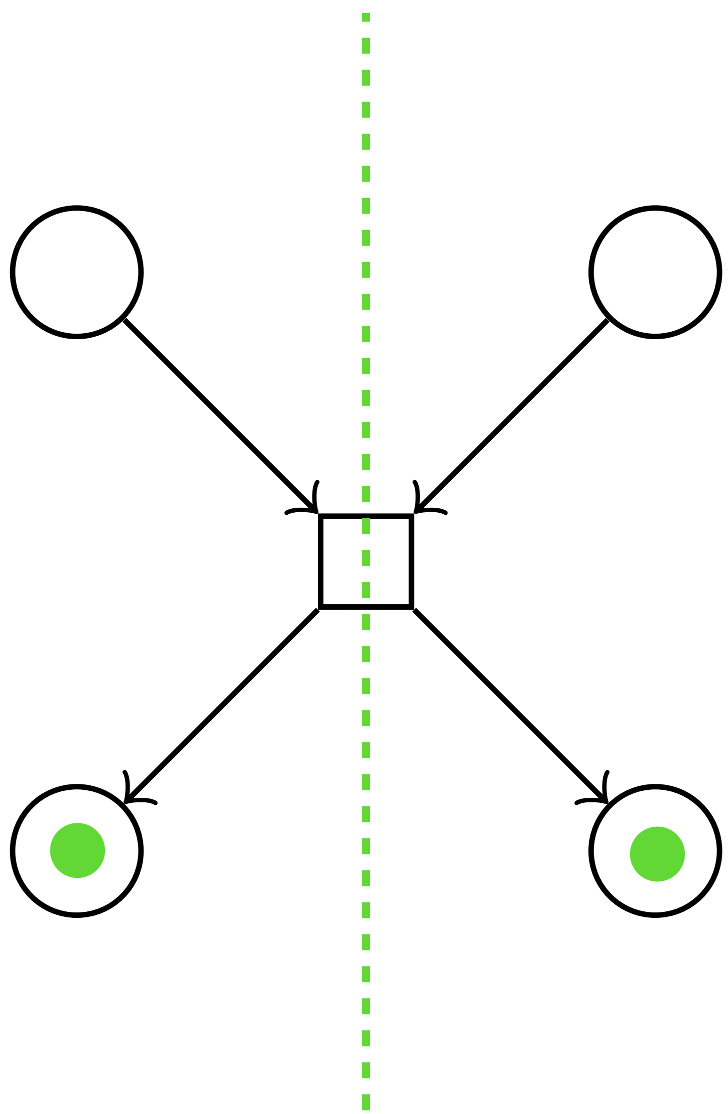

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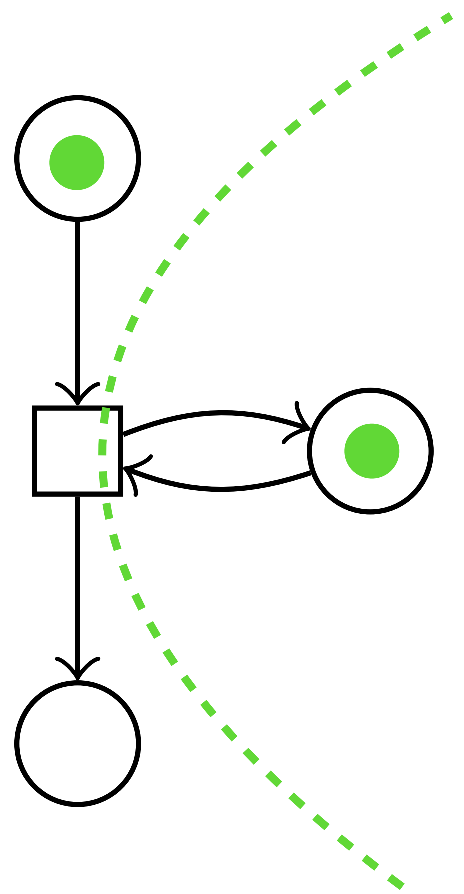

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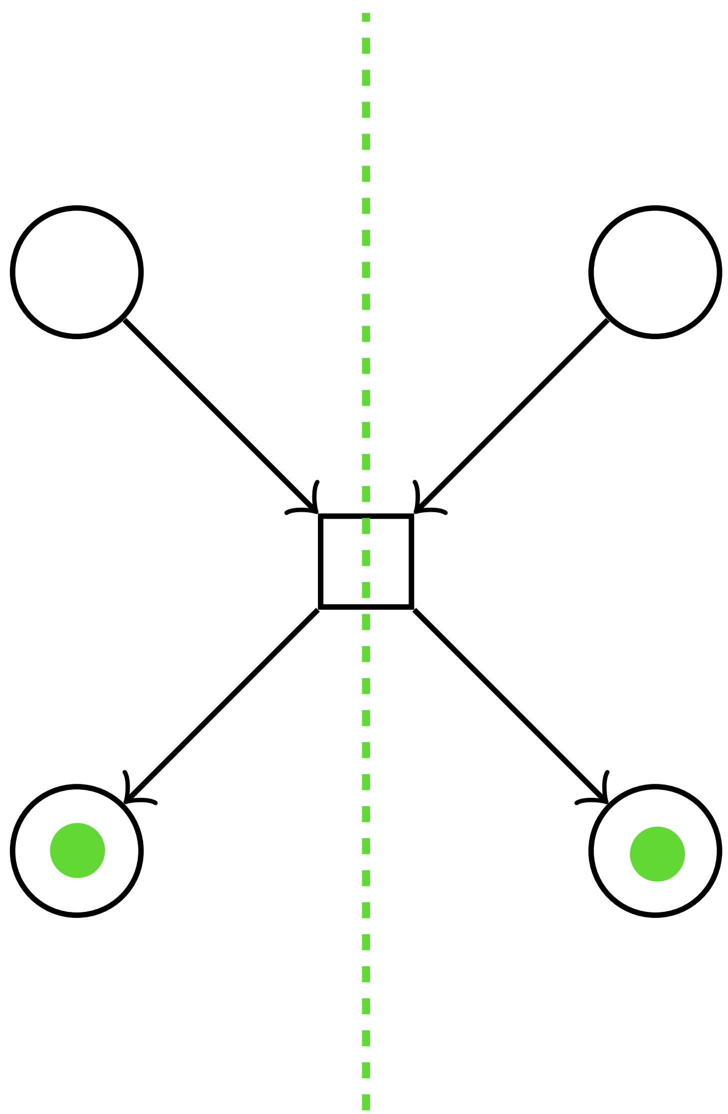

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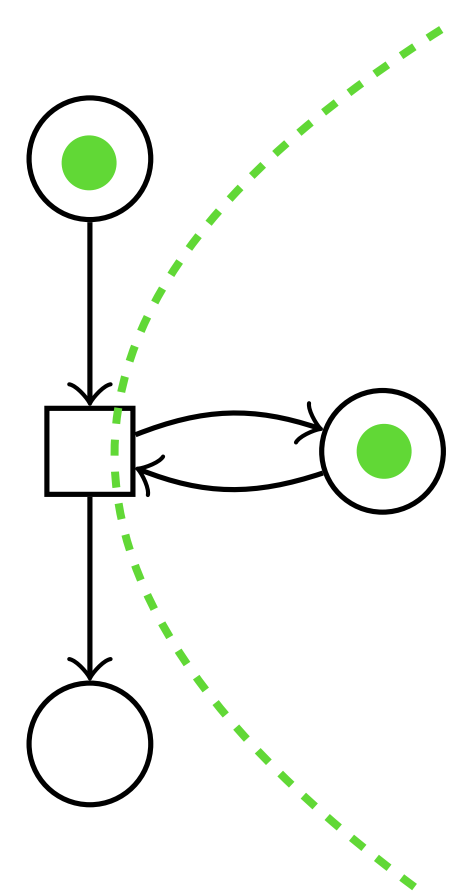

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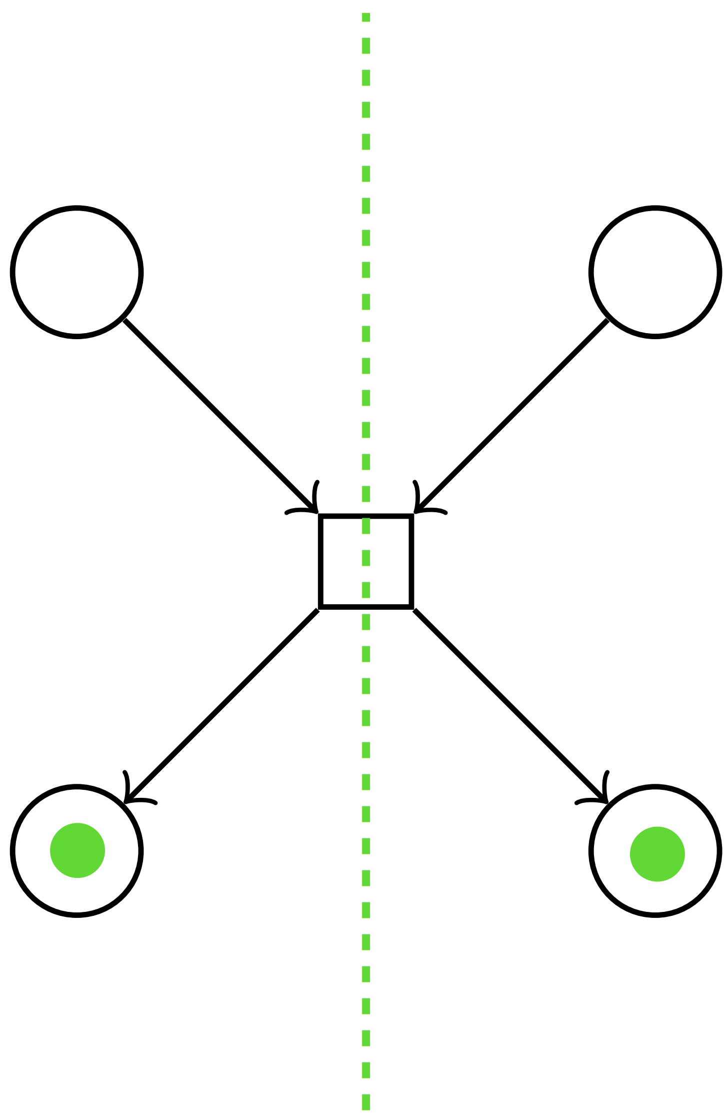

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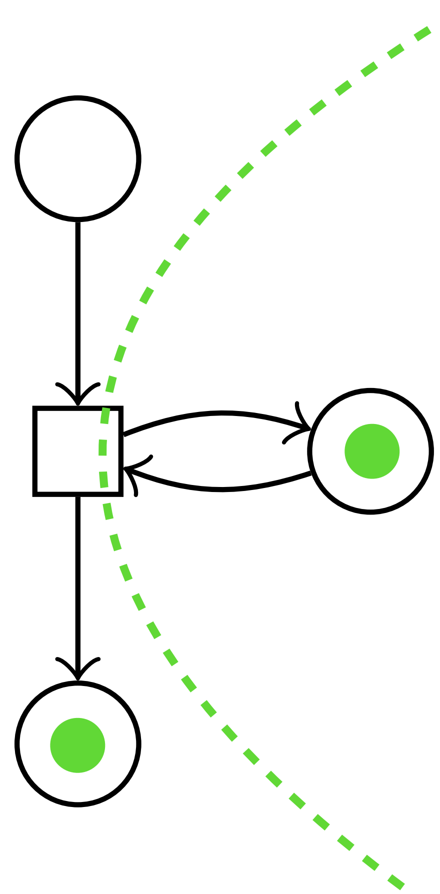

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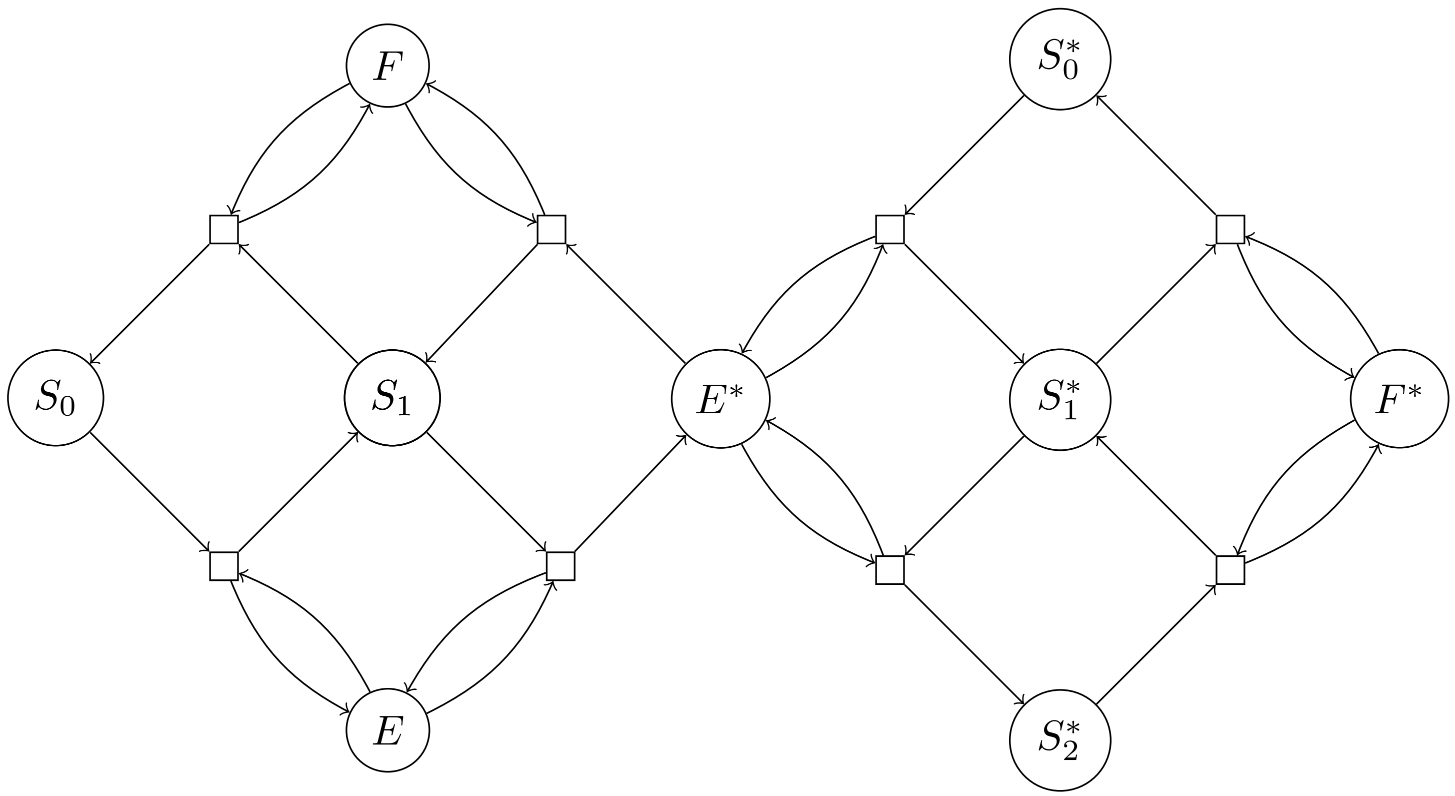
Application: Chemical Reaction Networks

enzyme → E, reactant → R, product → P

Enzymatic catalysis: $E + R \longrightarrow E + P$

$$E + S_0 \rightarrow E + S_1 \rightarrow E + E^*$$
$$F + E^* \rightarrow F + S_1 \rightarrow F + S_0$$
$$E^* + S_0^* \rightarrow E^* + S_1^* \rightarrow E^* + S_2^*$$
$$F^* + S_2^* \rightarrow F^* + S_1^* \rightarrow F^* + S_0^*$$



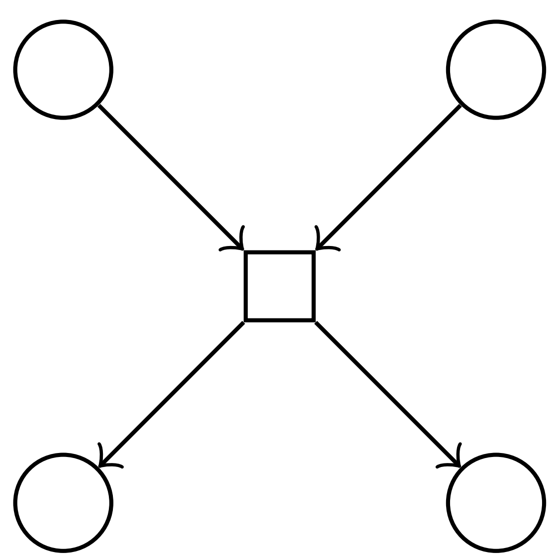
[A Petri net approach to the study of persistence in chemical reaction networks, Angeli et al., ’06]

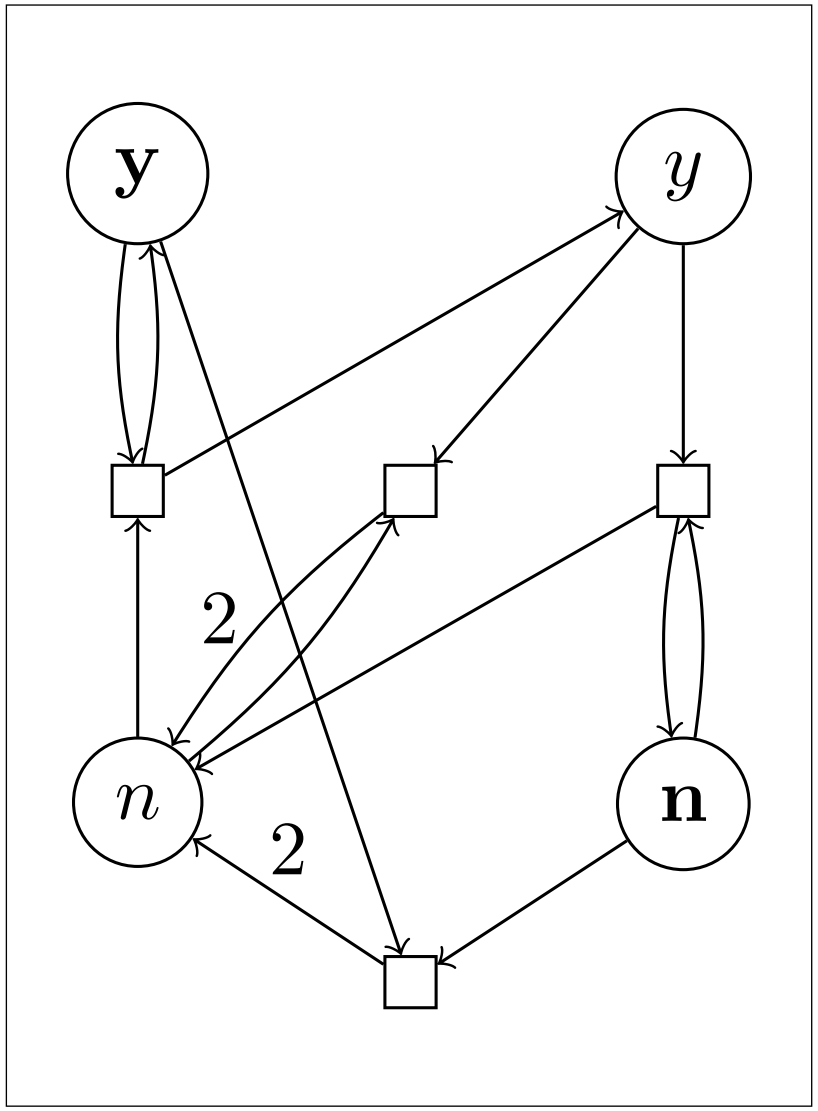
Application: Population Protocols

[Angluin et al.*, '04]*

Distributed computing model where identical finite-state mobile agents jointly compute a function.

Agents communicate through *rendez-vous*.





[The computational power of population protocols, Angluin et al.,'06]

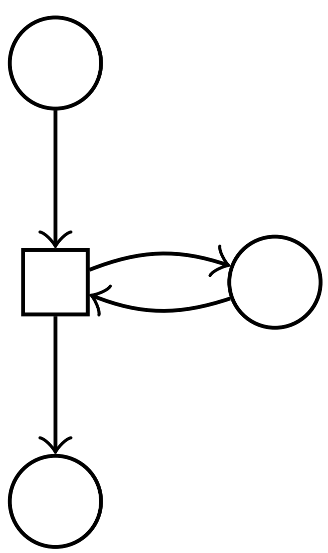
Application: Population Protocols

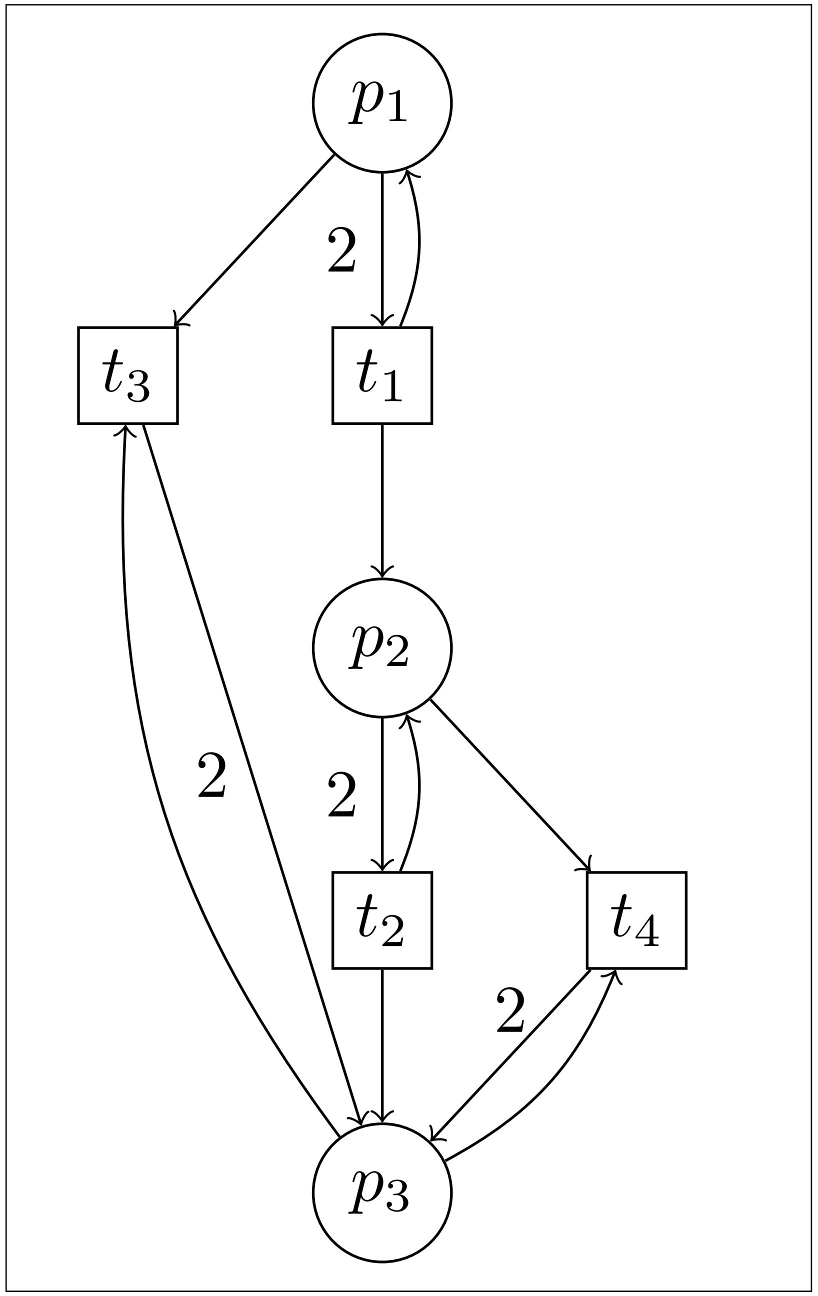
Distributed computing model for identical finite-state mobile agents.

Immediate observation population protocols:

An agent *observes* an other agent's state and updates its own based on this information.

Introduced to model sensor networks.





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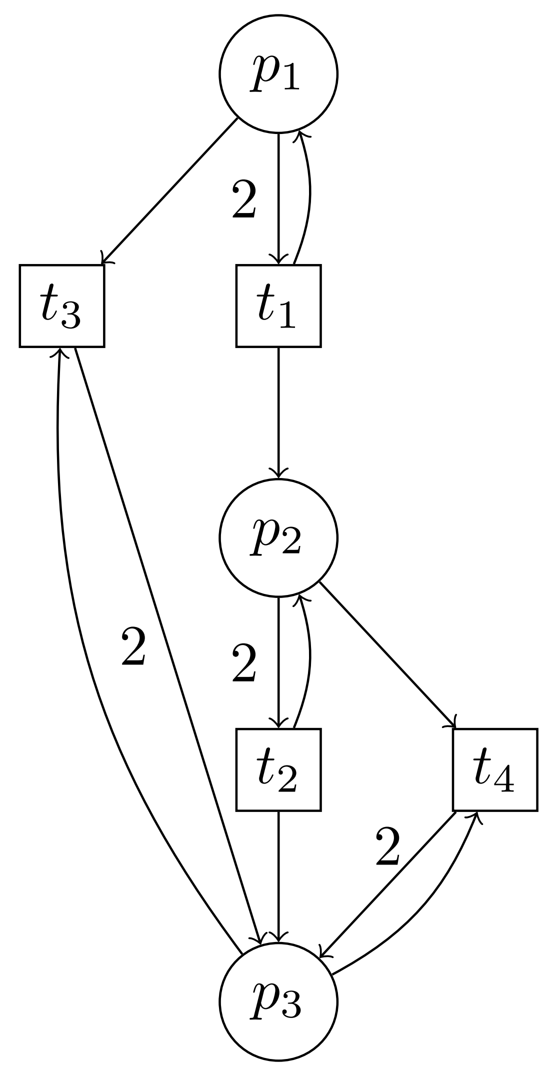
Parameterized Problems

In these application domains we are interested in *parameterized* problems.



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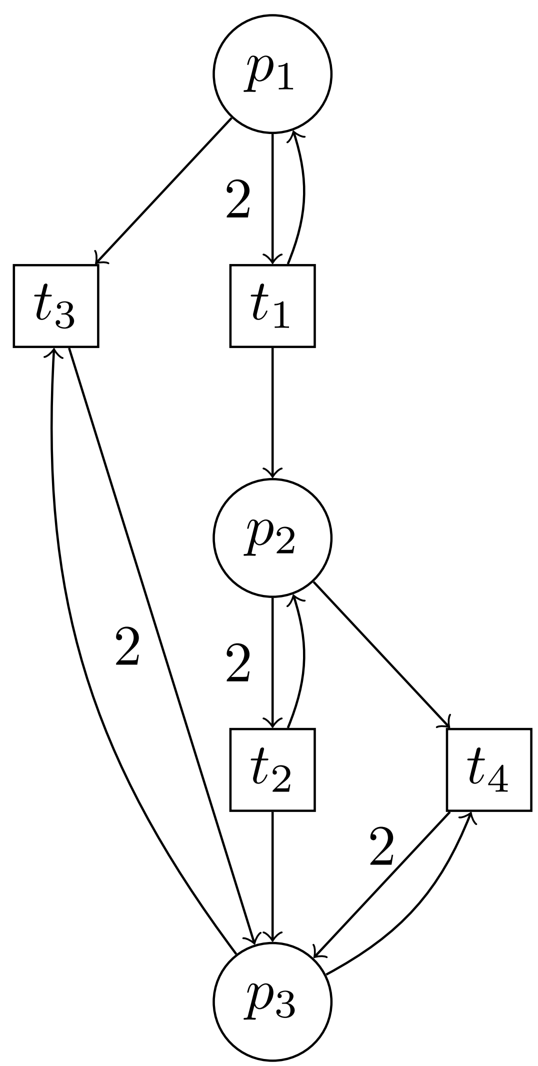
- Goal of a protocol: compute a function $f : \mathbb{N}^k \to \{0,1\}$
- Protocol: Petri net N. Input: initial marking M_0.
- Correctness: for every initial marking M_0, the Petri net (N, M_0) "computes" $f(M_0)$.



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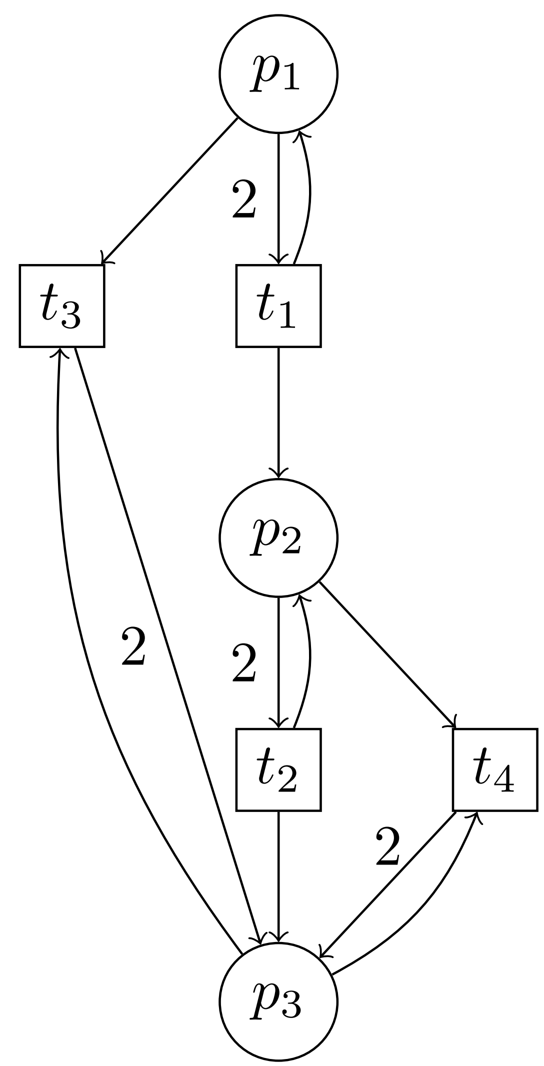

[Angluin et al.*,'06]*

Initial markings: $M_0^{(n)} = n \cdot p_1$

parameter

This protocol is correct if and only if for every initial marking $M_0^{(n)}$:

- $n \geq 3 \Rightarrow$ all markings reachable from $M_0^{(n)}$ can reach the marking with all tokens in p_3.
- $n < 3 \Rightarrow$ there is no reachable marking with a token in p_3.

Counting Constraints

We consider infinite sets of markings defined by counting constraints.

number of tokens in p_2

lower bound

upper bound

- An expression $2 \leq x_2 \leq 5$ is an *atomic bound.*

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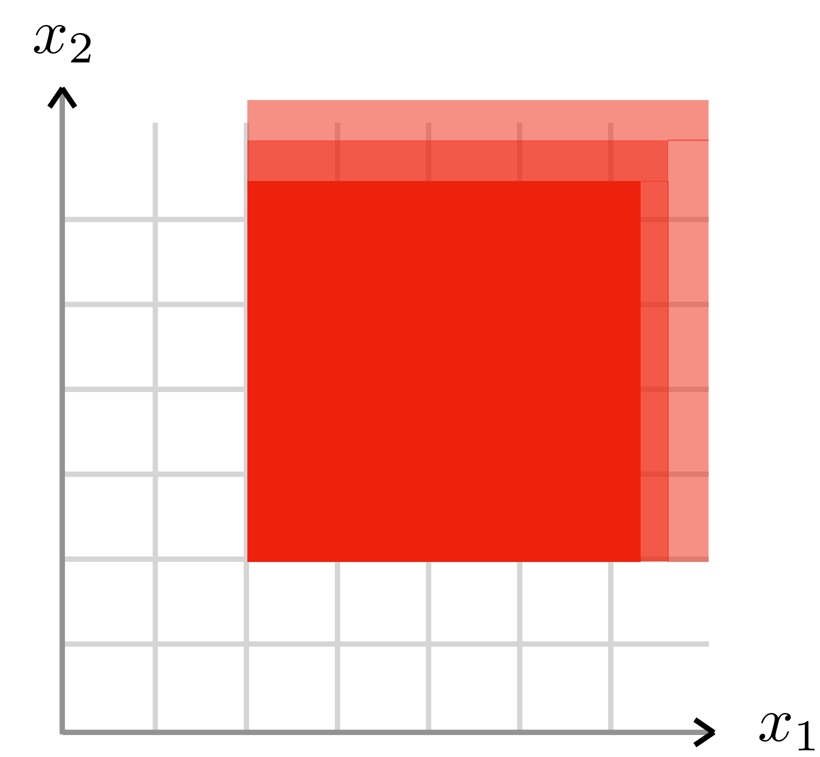
e.g. in a Petri net with two places p_1 and p_2



$2 \leq x_1 \leq \infty \wedge$
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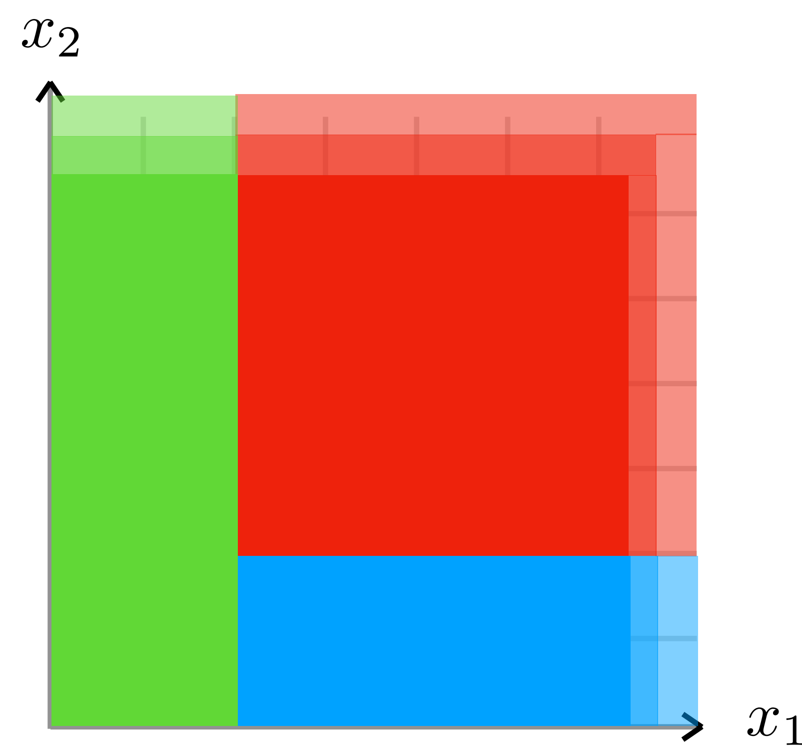

$$\neg \begin{array}{l} 2 \leq x_1 \leq \infty \wedge \\ 2 \leq x_2 \leq \infty \end{array}$$

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Parameterized Reachability and Coverability

INPUT: An IO net N, and two sets of markings S and S' described by **counting constraints**.

Parameterized Reachability

QUESTION: Are there markings $M \in S$ and $M' \in S'$ such that M' is **reachable** from M in net N ?

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Results

	Conservative Petri nets	**Immediate Observation nets**
Reachability	PSPACE-complete	
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Compared to arbitrary conservative Petri nets, IO nets

- don't lose expressivity, and
- do much better for parameterized problems: deciding for infinitely many markings is not harder than for a single marking!

Application: Correctness of IOPP

[Angluin, Aspnes, Diamada, Fischer, Peralta, '04]

General population protocols

[Angluin, Aspnes, Eisenstat, Ruppert, '07]

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↓

[This paper]

Correctness is **PSPACE-complete**

Main Theorem

Theorem

For *N* an IO net with *n* places, for Γ a counting constraint describing a set *S*,

1. there exist counting constraints for *pre*(S)* and *post*(S)*
2. the size of these counting constraints is $\leq size(\Gamma) + n^3$

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essentially the largest finite bound
of the counting constraint

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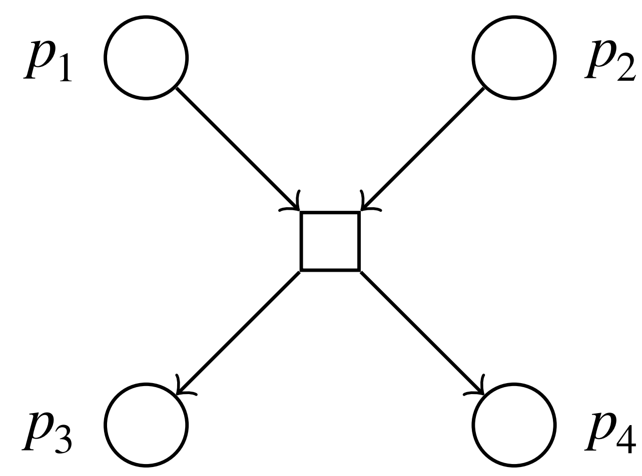

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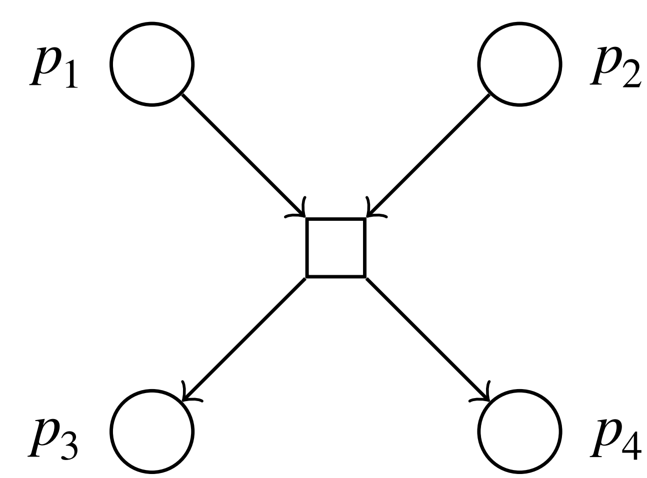

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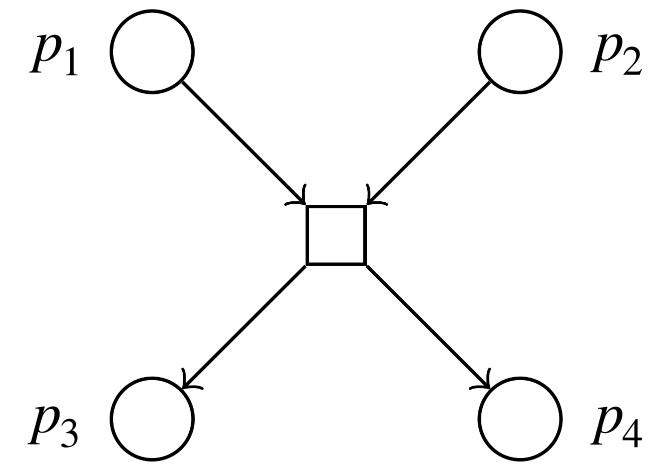

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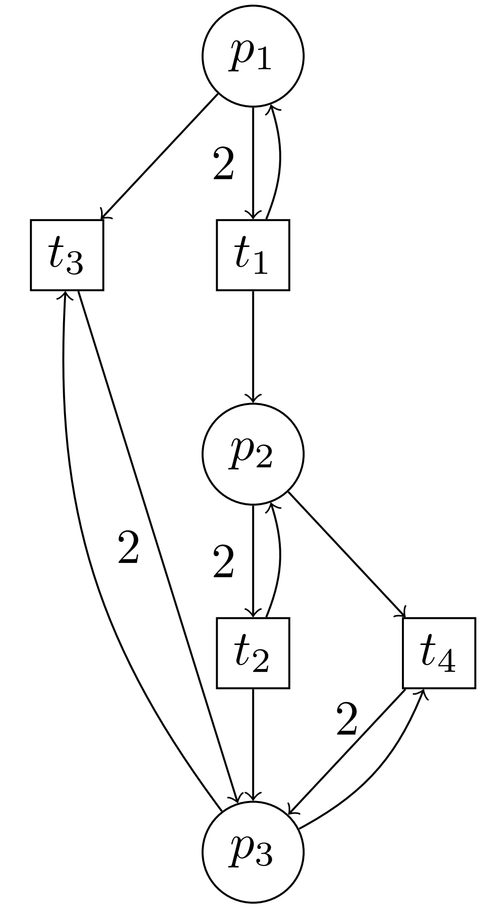
Not a counting constraint!

Applying the Main Theorem



By the Main Theorem, we know we can apply the following algorithm to counting constraint S to obtain $post^*(S)$.

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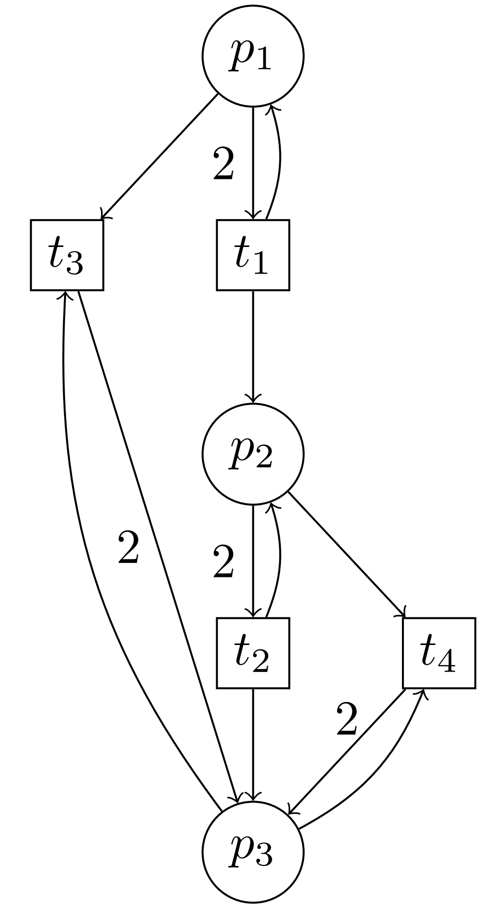

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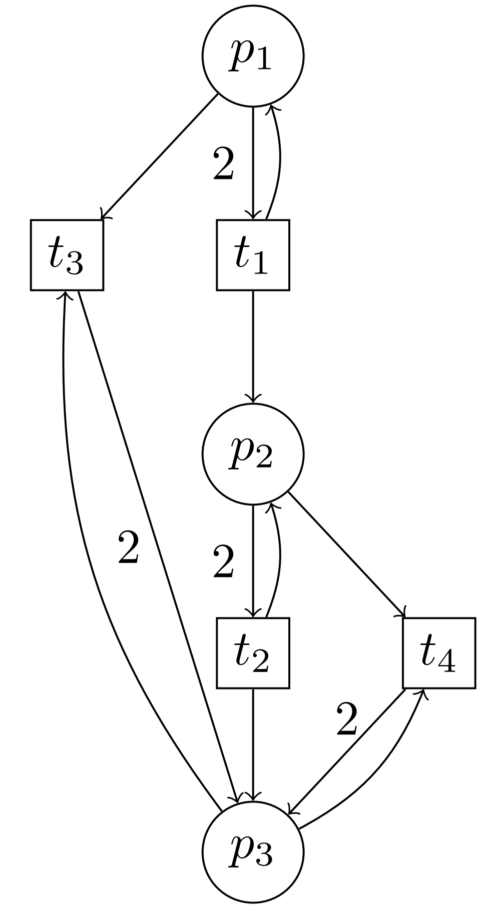

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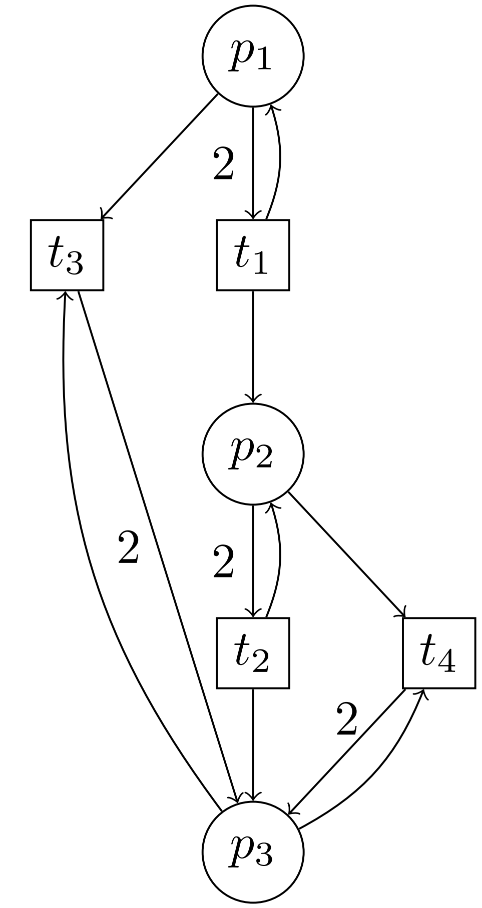

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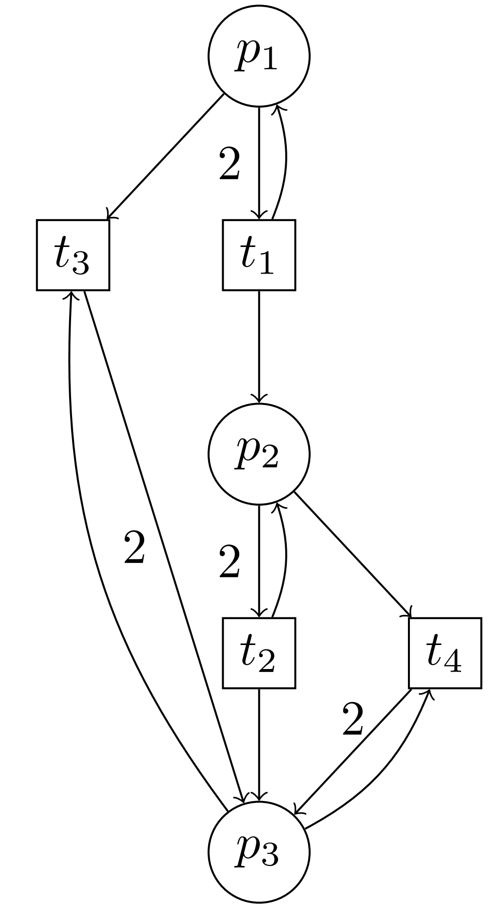

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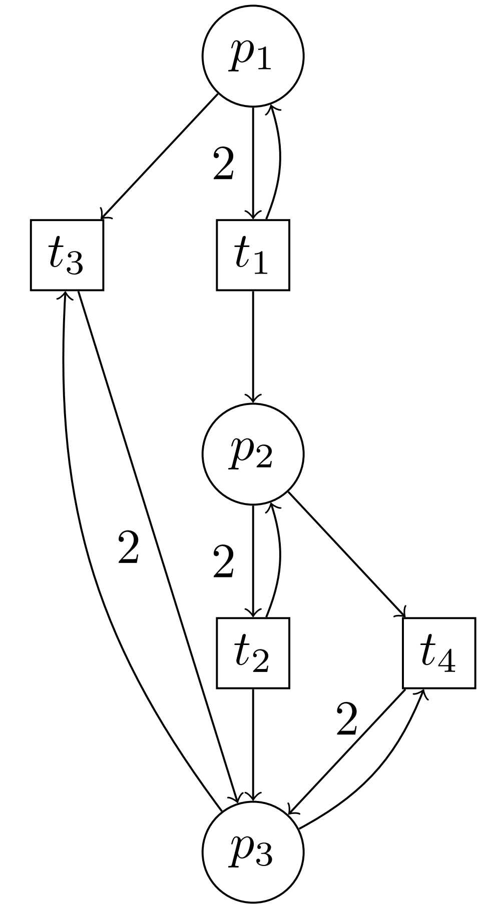

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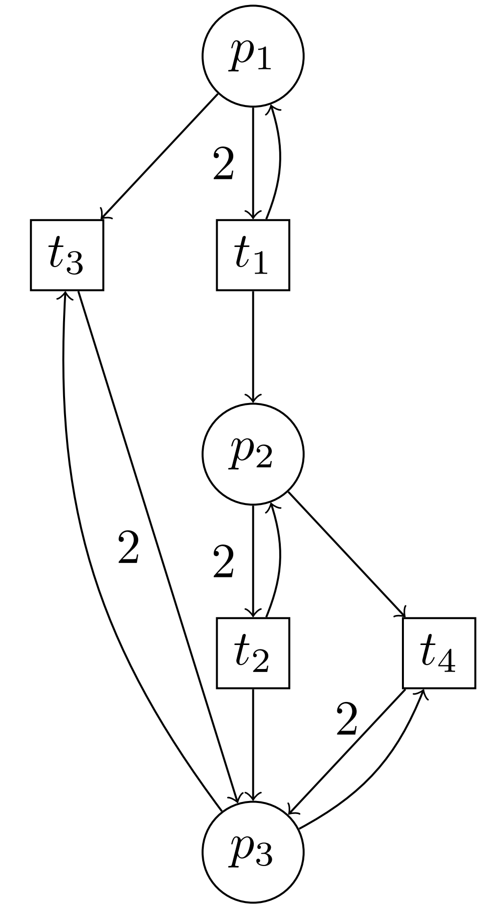

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$\xrightarrow{t_1^*}$

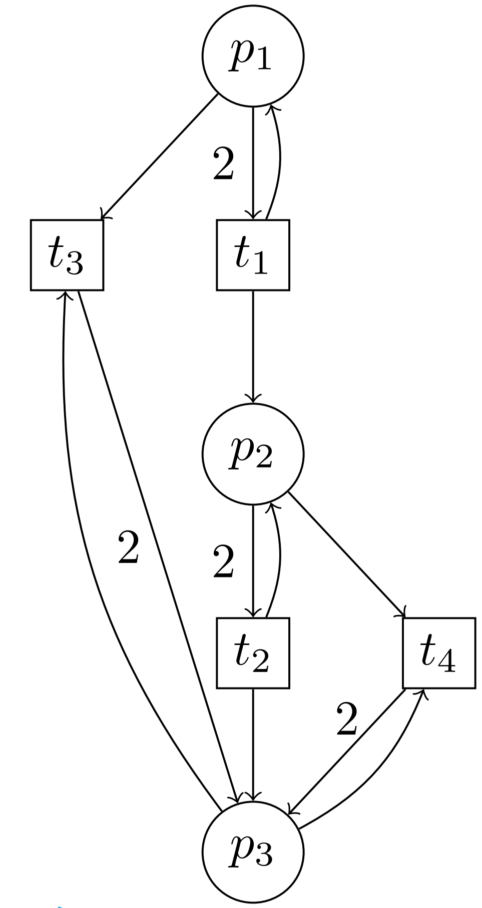
$$S_1: S_0 \vee \begin{pmatrix} 2 \leq p_1 \leq \infty \wedge \\ 1 \leq p_2 \leq \infty \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix} \vee \begin{pmatrix} 1 \leq p_1 \leq \infty \wedge \\ 2 \leq p_2 \leq \infty \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix}$$

$\xrightarrow{t_1^*}$ S_1

$\xrightarrow{t_2^*}$

$$S_2: S_1 \vee \begin{pmatrix} 1 \leq p_1 \leq \infty \wedge \\ 1 \leq p_2 \leq \infty \wedge \\ 1 \leq p_3 \leq \infty \end{pmatrix}$$

Applying the Main Theorem



By the Main Theorem, we know we can apply the following algorithm to counting constraint S to obtain $post^*(S)$.

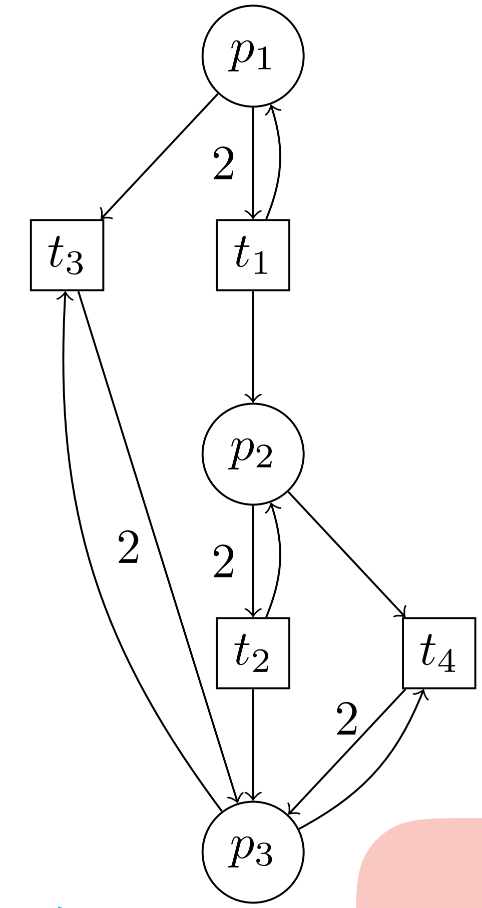
$$S = S_0 = \begin{pmatrix} 3 \leq p_1 \leq \infty \wedge \\ 0 \leq p_2 \leq 0 \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix} \xrightarrow{t_1^*} S_1 = S_0 \vee \begin{pmatrix} 2 \leq p_1 \leq \infty \wedge \\ 1 \leq p_2 \leq \infty \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix} \vee \begin{pmatrix} 1 \leq p_1 \leq \infty \wedge \\ 2 \leq p_2 \leq \infty \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix}$$

$$S_1 \xrightarrow{t_1^*} S_1 \quad (\xrightarrow{t_1^*} \cdots, \ \xrightarrow{t_2^*} \cdots)$$

$$S_1 \xrightarrow{t_2^*} S_2 = S_1 \vee \begin{pmatrix} 1 \leq p_1 \leq \infty \wedge \\ 1 \leq p_2 \leq \infty \wedge \\ 1 \leq p_3 \leq \infty \end{pmatrix} \quad (\xrightarrow{t_1^*}, \xrightarrow{t_2^*}, \xrightarrow{t_3^*} \cdots)$$

$S = S_0$ $\qquad S_1$ $\qquad S_2$

Applying the Main Theorem



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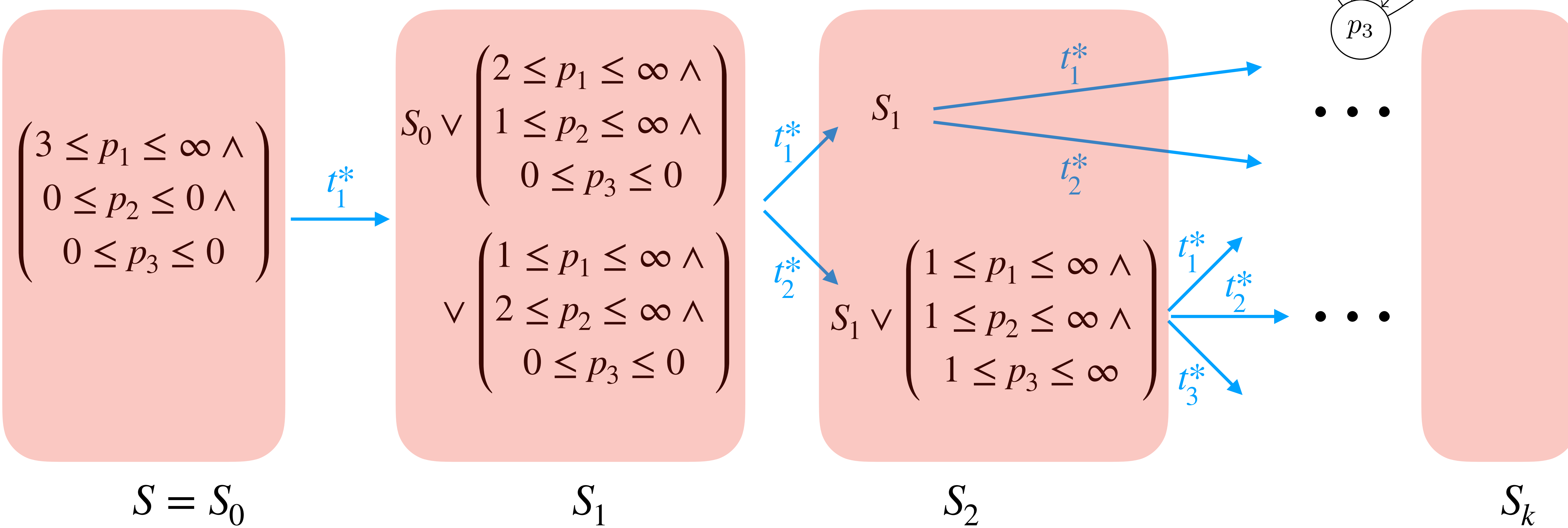
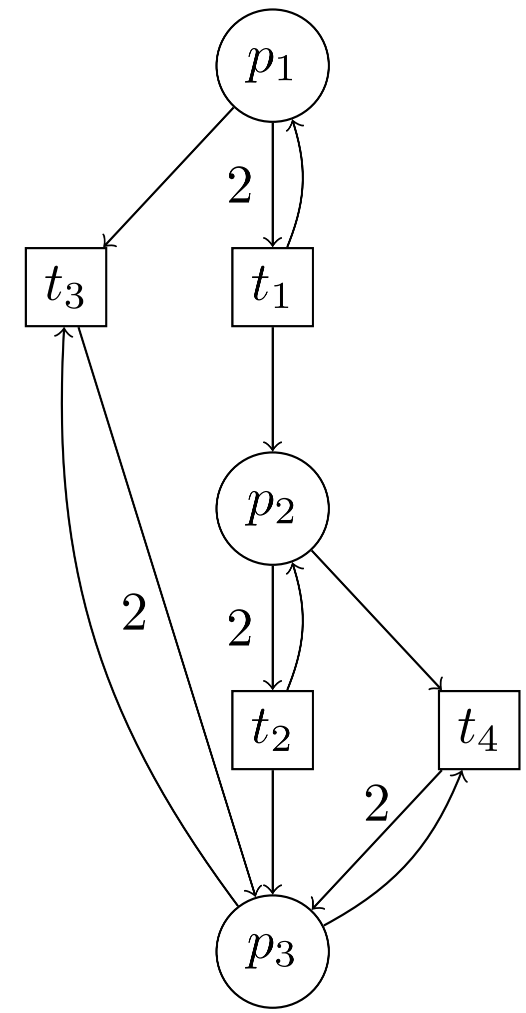


$$post^*(S) = S_0 \vee S_1 \vee S_2 \vee \ldots \vee S_k$$

Applying the Main Theorem

This protocol is correct if and only if for every initial marking $M_0^{(n)}$:

- $n \geq 3 \Rightarrow$ all markings reachable from $M_0^{(n)}$ can reach the marking with all tokens in p_3.
- $n < 3 \Rightarrow$ there is no reachable marking with a token in p_3.



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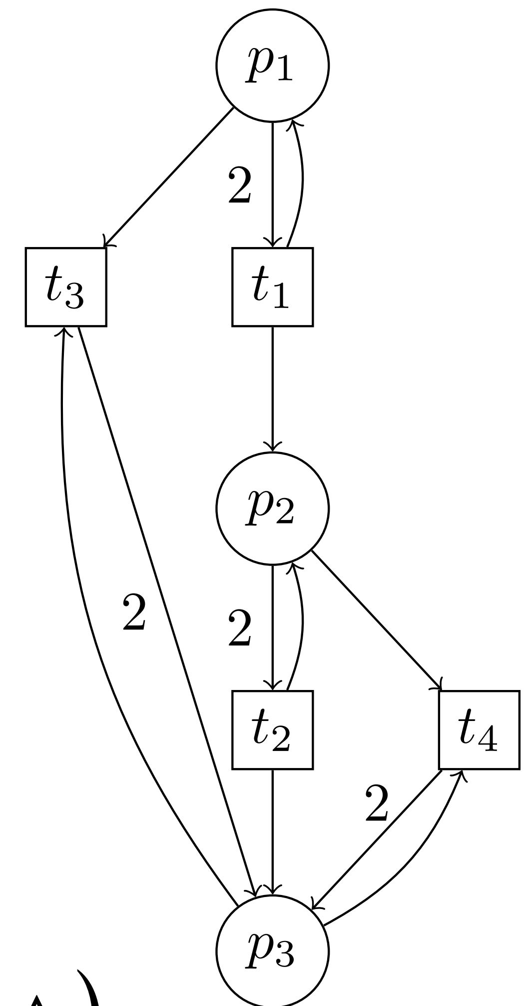

$$post^*\begin{pmatrix} 3 \leq p_1 \leq \infty \wedge \\ 0 \leq p_2 \leq 0 \wedge \\ 0 \leq p_3 \leq 0 \end{pmatrix} \subseteq pre^*\begin{pmatrix} 0 \leq p_1 \leq 0 \wedge \\ 0 \leq p_2 \leq 0 \wedge \\ 3 \leq p_3 \leq \infty \end{pmatrix}$$

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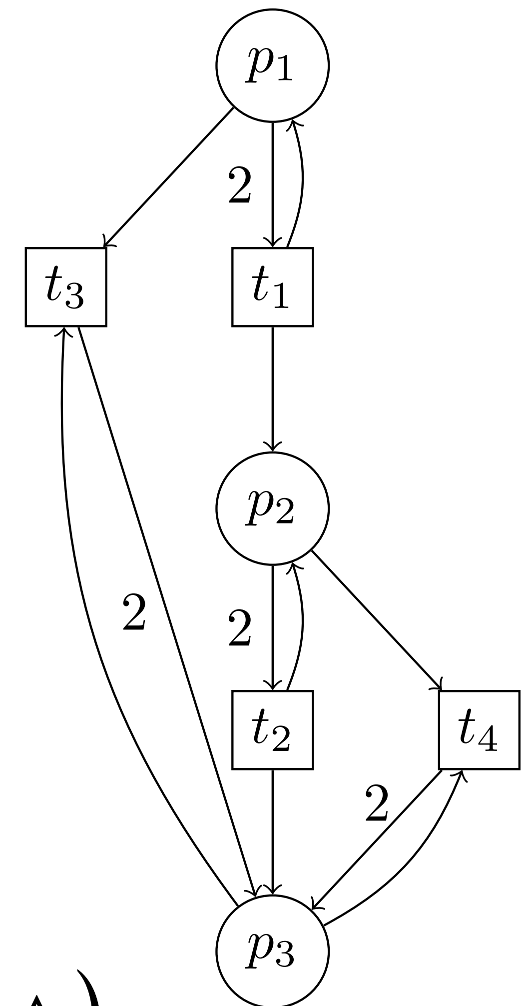

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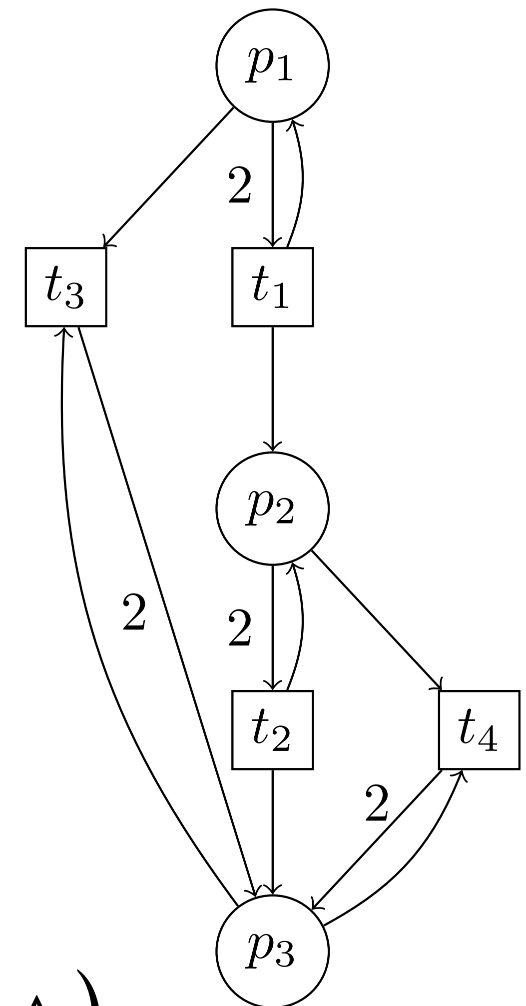

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check inclusion

Conclusion

- We study class of Petri nets with applications to population protocols and chemical reaction networks.

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Thank you !

Building Block

Pruning Theorem

Let *N* be an IO net with *n* places.

If a marking *M’* is coverable by some marking *M*, then *M’* is coverable by some marking *C* such that

1. *C* is covered by *M*.
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Pruning: we remove tokens from the run that covers M' without modifying its covering property.