On Interval Semantics of Inhibitor and Activator Nets

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Petri Nets 2019

Inhibitor and Activator Nets

- *Inhibitor nets* are Petri nets enriched with *inhibitor arcs* and *activator nets* are Petri nets enriched with *activator arcs*.
- *Inhibitor arcs* allow a transition to check for an *absence* of a token. In principle they allow 'test for zero', an operator the standard Petri nets do not have.
- *Activator arcs* (also called 'read', or 'contextual' arcs), are conceptually orthogonal to the inhibitor arcs, they allow a transition to check for a *presence* of a token.
- *Elementary inhibitor nets* are just classical *elementary nets*, i.e. one-safe place-transition nets without self-loops, extended with *inhibitor arcs*.
- Nevertheless they can easily express complex behaviours involving 'not later than' cases, priorities, various versions of simultaneities, etc.
- Similarly *elementary activator nets* are just classical elementary nets extended with *activator arcs*.

Operational Semantics and Expressiveness

- **Operational Semantics**:
  - *Firing sequences*, i.e. total orders,
  - *Firing step sequences*, i.e. stratified orders,
  - *Firing interval sequences*, i.e. interval orders,

  The first two are well known, the third one is much less developed and understood.
- **Firing sequences:** both inhibitor and activator elementary nets can be represented by equivalent one-safe nets with self-loops - **not true** if simultaneous executions, for instance steps, are allowed.
- **Firing sequences and firing step sequences:** each elementary net with inhibitor arcs can always equivalently be represented by an appropriate elementary net with activator arcs - **not true** for **firing interval sequences (this paper)**.

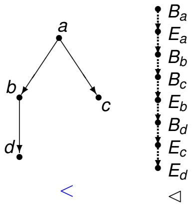
Interval Orders

Theorem (Fishburn 1970)

A (discrete) partial order $<$ on countable set X is interval iff there exists a total order $\lhd$ on some T and two (injective) mappings with (disjoint) codomains $B, E : X \to T$ such that for all $x, y \in X$,

1. $B(x) \lhd E(x)$ 2. $x < y \iff E(x) \lhd B(y)$

We will write B_x, E_x instead of $B(x), E(x)$.



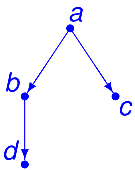
A well defined sequence of B_a's and E_a's is called an **interval sequence**.

Notation: The set of all interval sequences will be denoted by IntSeq.

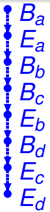
For every *interval sequence* x, $\blacktriangleleft_x$ is the *interval order* defined by x via Fishburn Theorem.

For example, for $x = B_aE_aB_bB_cE_bB_dE_cE_d$, $\blacktriangleleft_x$ is the partial order $<$ above.

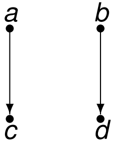
Partial Orders



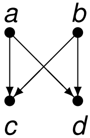
$<_1$



$\lhd_1$



$<_2$



$<_3$

- The order $<_1$ is interval but not stratified.
- The order $<_2$ is not interval.
- The order $<_3$ is stratified.
- The total order $\lhd_1$ is one of Fishburn's representations of the interval order $<_1$. It has the *interval sequence* representation $x_1 = B_aE_aB_bB_cE_bB_dE_cE_d$.
- However the interval sequences $x_2 = B_aE_aB_cB_bE_bB_dE_cE_d$, $x_3 = B_aE_aB_bB_cE_bB_dE_dE_c$, $x_4 = B_aE_aB_cB_bE_bB_dE_dE_c$ also represent the interval order $<_1$, i.e. $\blacktriangleleft_{x_1} = \blacktriangleleft_{x_2} = \blacktriangleleft_{x_3} = \blacktriangleleft_{x_4} = <_1$.
- A stratified order $<_3$ is represented by a step sequence $\{a, b\}\{c, d\}$.

Sound Interval Sequence Operational Semantics

- Let $\mathcal{M}$ be a model of concurrent system (i.e. Petri net, process algebra expressions, some automaton, etc.) that allows to define its operational semantics in terms of interval sequences, and issem($\mathcal{M}$) be the set of interval sequences that describes the operational semantics of $\mathcal{M}$.
- we may define the interval order operational semantics

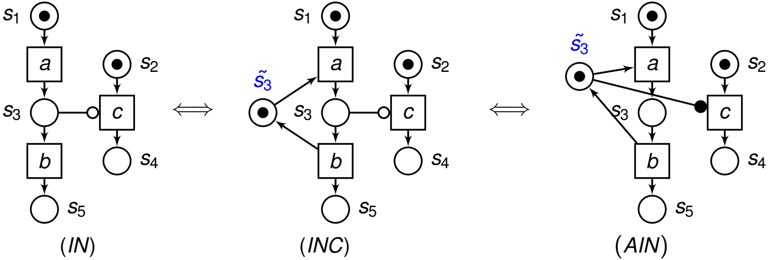
$$\text{IOSEM}(\mathcal{M}) = \{\blacktriangleleft_x \mid x \in \text{issem}(\mathcal{M})\}.$$

- A model $\mathcal{M}$ has a **sound interval operational semantics** iff:

$$\{z \in \text{IntSeq} \mid \blacktriangleleft_z \in \text{IOSEM}(\mathcal{M})\} = \text{issem}(\mathcal{M}).$$

- If an interval sequence operational semantics of $\mathcal{M}$ is not sound, issem($\mathcal{M}$) may still be valid concept, but we have to use IOSEM($\mathcal{M}$) very carefully, as it may not be a valid construction.

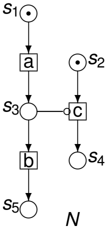
Inhibitor vs Activator Nets: Sequences and Step Sequences

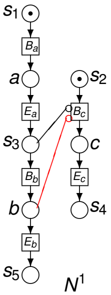


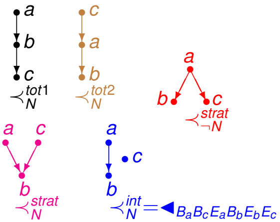
- The net *IN* is an inhibitor net but it is **not** complement closed. Adding the place $\widetilde{s}_3$ makes it complement closed and transforms it into the net *INC*.
- The net *AIN* was derived from *INC* by replacing the inhibitor arc (s_3, c) with the activator arc $(\widetilde{s}_3, c)$.
- All three nets generate exactly the same set of *firing sequences* and *firing step sequences* (but **not** *interval firing sequences*!).

Interval Sequence (Interval Order) Semantics of Inhibitor Nets

Notation: FS/FSS/FIS$_N(m \rightsquigarrow m')$ denote respectively Firing Sequences/Firing Step Sequences/Firing Interval Sequences of the net N from marking m to marking m', and let $m_0 = \{s_1, s_2\}$, $m_F = \{s_4, s_5\}$.



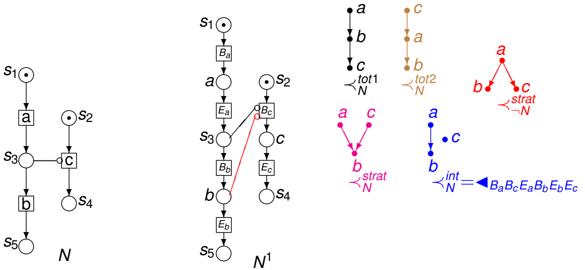




$\text{FIS}_N(m_0 \rightsquigarrow m_F) \stackrel{df}{=} \text{FS}_{N^1}(m_0 \rightsquigarrow m_F) = \{B_aE_aB_bE_bB_cE_c, B_cE_cB_aE_aB_bE_b,$
$B_aB_cE_aE_cB_bE_b, B_aB_cE_cE_aB_bE_b, B_cB_aE_aE_cB_bE_b, B_cB_aE_cE_aB_bE_b,$
$B_aB_cE_aB_bE_bE_c, B_aB_cE_aB_bE_cE_b, B_cB_aE_aB_bE_bE_c, B_cB_aE_aB_bE_cE_b\}$

- There is no sequence x in $\text{FS}_{N^1}(m_0 \rightsquigarrow m_F)$ such that $\prec_{\neg IN}^{strat} = \blacktriangleleft_x$, however if the red inhibitor arc is deleted, the new net has for example a firing sequence $B_aE_aB_bB_cE_bEc$ and $\prec_{\neg IN}^{strat} = \blacktriangleleft_{B_aE_aB_bB_cE_bEc}$.

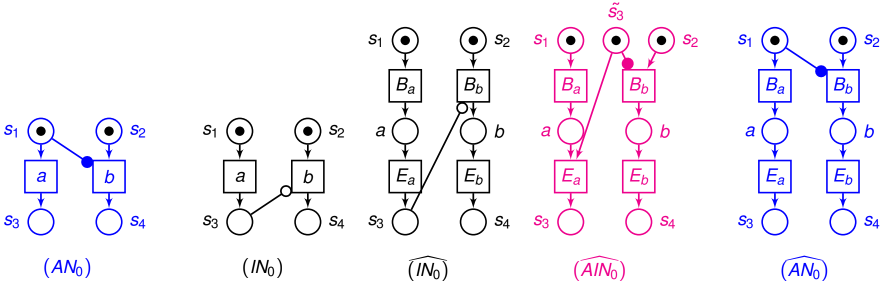
Notation: $IO/SO_N(m \rightsquigarrow m')$ denote respectively Interval Orders/Stratified Orders generated by the net N from marking m to marking m', STRAT denotes all stratified orders and let $m_0 = \{s_1, s_2\}$, $m_F = \{s_4, s_5\}$.



$$FIS_N(m_0 \rightsquigarrow m_F) \overset{df}{=} FS_{N^1}(m_0 \rightsquigarrow m_F) = \{B_aE_aB_bE_bB_cE_c, B_cE_cB_aE_aB_bE_b,$$
$$B_aB_cE_aE_cB_bE_b, B_aB_cE_cE_aB_bE_b, B_cB_aE_aE_cB_bE_b, B_cB_aE_cE_aB_bE_b,$$
$$B_aB_cE_aB_bE_bE_c, B_aB_cE_aB_bE_cE_b, B_cB_aE_aB_bE_bE_c, B_cB_aE_aB_bE_cE_b\}$$
$$FSS_N(m_0 \rightsquigarrow m_f) = \{\{a\}\{b\}\{c\}, \{c\}\{a\}\{b\}, \{a,c\}\{b\}\}$$
$$IO_{N^1}(m_0 \rightsquigarrow m_f) = \{\prec^{tot1}_N, \prec^{tot2}_N, \prec^{strat}_N, \prec^{int}_N\}.$$
$$SO_N(m_0 \rightsquigarrow m_f) = \{\prec^{tot1}_N, \prec^{tot2}_N, \prec^{strat}_N\} = IO_{N^1}(m_0 \rightsquigarrow m_f) \cap \text{STRAT}.$$

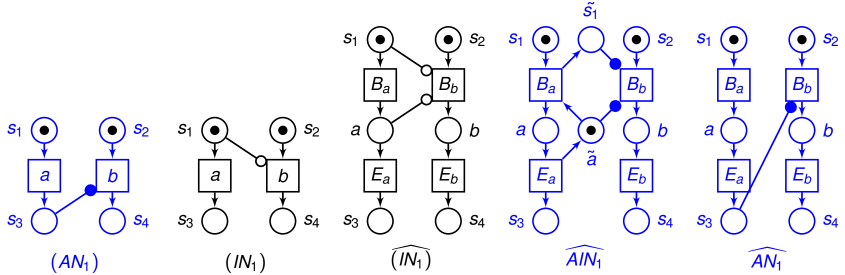
- Interval Sequence Semantics is **sound**. (Proposition 7)
- Interval Sequence Semantics is **consistent** with Step Sequence Semantics. (Proposition 8)

Notation: FS/FSS/FIS$_N(m \rightsquigarrow m')$ denote respectively Firing Sequences/Firing Step Sequences/Firing Interval Sequences of the net N from marking m to marking m' and let $m_0 = \{s_1, s_2\}$, $m_F = \{s_3, s_4\}$.



Clearly $\text{FS}_{AN_0}(m_0 \rightsquigarrow m_F) = \text{FS}_{IN_0}(m_0 \rightsquigarrow m_F) = \{ba\}$,
$\text{FSS}_{AN_0}(m_0 \rightsquigarrow m_F) = \text{FSS}_{IN_0}(m_0 \rightsquigarrow m_F) = \{\{b\}\{a\}, \{a, b\}\}$, and
$\text{FIS}_{IN_0}(m_0 \rightsquigarrow m_F) \overset{df}{=} \text{FS}_{\widehat{IN_0}}(m_0 \rightsquigarrow m_F) = \text{FS}_{\widehat{AIN_0}}(m_0 \rightsquigarrow m_F) =$
$\{B_bE_bB_aE_a, B_aB_bE_aE_b, B_aB_bE_bE_a, B_bB_aE_aE_b, B_bB_aE_bE_a\}$.

However:
$\text{FIS}_{AN_0}(m_0 \rightsquigarrow m_F) \overset{df}{=} \text{FS}_{\widehat{AN_0}}(m_0 \rightsquigarrow m_F) = \{B_bE_bB_aE_a, B_bB_aE_aE_b, B_bB_aE_bE_a\} \subsetneq$
$\text{FS}_{\widehat{IN_0}}(m_0 \rightsquigarrow m_F) = \text{FIS}_{IN_0}(m_0 \rightsquigarrow m_F)$.

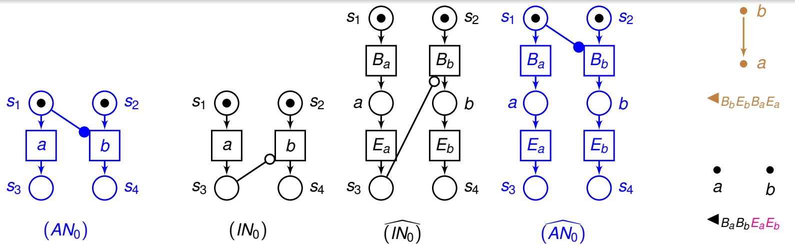


Assume $m_0 = \{s_1, s_2\}, m_F = \{s_3, s_4\}$. Clearly
$FS_{AN_1}(m_0 \rightsquigarrow m_F) = FS_{IN_1}(m_0 \rightsquigarrow m_F) = \{ab\}$,
$FSS_{AN_1}(m_0 \rightsquigarrow m_F) = FSS_{IN_1}(m_0 \rightsquigarrow m_F) = \{\{a\}\{b\}\}$, and

$FIS_{IN_1}(m_0 \rightsquigarrow m_F) \stackrel{df}{=}$
$FS_{\widehat{IN_1}}(m_0 \rightsquigarrow m_F) = \{B_aE_aB_bE_b\}$.

Moreover, $FIS_{IN_1}(m_0 \rightsquigarrow m_F) = FS_{\widehat{IN_1}}(m_0 \rightsquigarrow m_F) = FS_{\widehat{AIN_1}}(m_0 \rightsquigarrow m_F) =$
$FS_{\widehat{AN_1}}(m_0 \rightsquigarrow m_F) = \{B_aE_aB_bE_b\}$.

Hence $FIS_{AN_1}(m_0 \rightsquigarrow m_F) \stackrel{df}{=} FS_{\widehat{AN_1}}(m_0 \rightsquigarrow m_F)$ is OK.



$\mathrm{FIS}_{IN_0}(m_0 \rightsquigarrow m_F) \stackrel{df}{=} \mathrm{FS}_{\widehat{IN_0}}(m_0 \rightsquigarrow m_F) =$
$\{B_bE_bB_aE_a, B_aB_bE_aE_b, B_aB_bE_bE_a, B_bB_aE_aE_b, B_bB_aE_bE_a\}$.

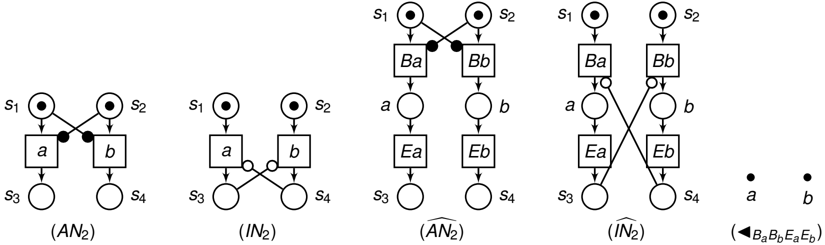
$\mathrm{FIS}_{AN_0}(m_0 \rightsquigarrow m_F) \stackrel{df}{=} \mathrm{FS}_{\widehat{AN_0}}(m_0 \rightsquigarrow m_F) = \{B_bE_bB_aE_a, B_bB_aE_aE_b, B_bB_aE_bE_a\}$.

$\mathrm{IO}_{\widehat{IN_0}}(m_0 \rightsquigarrow m_F) = \{◀_{B_bE_bB_aE_a}, ◀_{B_aB_bE_aE_b}\} = \mathrm{IO}_{\widehat{AN_0}}(m_0 \rightsquigarrow m_F)$.

However, $B_aB_bE_aE_b, B_aB_bE_bE_a \notin \mathrm{FIS}_{AN_0}(m_0 \rightsquigarrow m_F)$ while
$◀_{B_aB_bE_aE_b} = ◀_{B_aB_bE_bE_a} = ◀_{B_aB_bE_aE_b}$.

- **Interval Sequence Operational Semantics of Activator Nets is NOT Sound.**
- There is no activator net AN such that $\mathrm{FIS}_{AN}(m_0 \rightsquigarrow m_F) =$ $\{B_bE_bB_aE_a, B_aB_bE_aE_b, B_aB_bE_bE_a, B_bB_aE_aE_b, B_bB_aE_bE_a\}$.

Let $m_0 = \{s_1, s_2\}, m_F = \{s_3, s_4\}$.



$\mathrm{FSS}_{AN_2}(m_0 \rightsquigarrow m_F) = \mathrm{FSS}_{IN_2}(m_0 \rightsquigarrow m_F) = \{\{a, b\}\}$,

$\mathrm{FIS}_{IN_2}(m_0 \rightsquigarrow m_F) \overset{df}{=} \mathrm{FS}_{\widehat{IN_2}}(m_0 \rightsquigarrow m_F) = \{B_aB_bE_aE_b, B_bB_aE_aE_b, B_aB_bE_bE_a, B_bB_aE_bE_a\}$,

$\mathrm{FIS}_{AN_2}(m_0 \rightsquigarrow m_F) \overset{df}{=} \mathrm{FS}_{\widehat{AN_2}}(m_0 \rightsquigarrow m_F) = \emptyset$.

$\mathrm{SO}_{AN_2}(m_0 \rightsquigarrow m_F) = \mathrm{SO}_{IN_2}(m_0 \rightsquigarrow m_F) = \{\blacktriangleleft_{B_aB_bE_aE_b}\}$,

$\mathrm{IO}_{IN_2}(m_0 \rightsquigarrow m_F) = \{\blacktriangleleft_{B_aB_bE_aE_b}\}$, but $\mathrm{IO}_{AN_2}(m_0 \rightsquigarrow m_F) = \emptyset$

Hence $\mathrm{SO}_{AN_2}(m_0 \rightsquigarrow m) \neq \mathrm{IO}_{AN_2}(m_0 \rightsquigarrow m) \cap \mathrm{STRAT}$.

- **The step sequence semantics of activator nets and the interval sequence semantics of activator nets are NOT consistent.**
- The standard firing sequence semantics of activator nets and the interval sequence semantics of activator nets are consistent (Lemma 11).

Simultaneity (Steps) of Instantaneous Events

- If we assume standard continues real time, then from the time-energy uncertainty relations

$$\Delta t \Delta E \geq \frac{\hbar}{2\pi},$$

  where t denotes time, E denotes energy and $\hbar$ is Planck's constant, we must conclude that simultaneous execution of instantaneous events is unobservable as it would require infinite energy ($\Delta t = 0$).
- For discrete time, often assumed in computation theory, we have $\Delta t > 0$ so the assumption that we can observe simultaneity of instantaneous events might be valid.

Simultaneity (Steps) of Instantaneous Events: Discrete Time

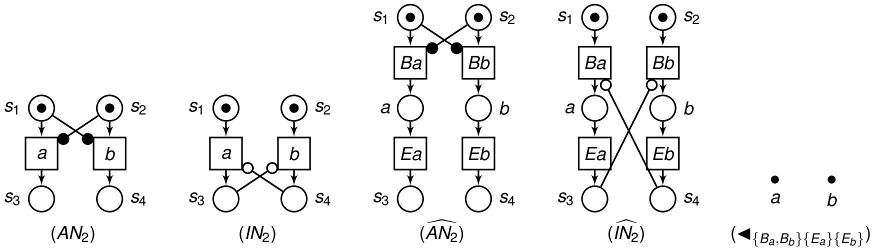
- If interval sequence operational semantics (or its equivalent) is sound, i.e.

$$\{z \in \text{IntSeq} \mid \blacktriangleleft_z \in \text{IOSEM}(\mathcal{M})\} = \text{issem}(\mathcal{M}),$$

  then simultaneous execution of instantaneous events is irrelevant as each step $\{B_a, B_b\}$ is equivalently represented by sequences $B_a B_b$ and $B_b B_a$, and similarly for other pairs and bigger steps.
- Hence, in terms of interval orders, interval sequence semantics and interval step sequence semantics would be equivalent, so why use the more complex one?
- The situation is much different when interval sequence operational semantics is not sound, as for instance the one defined in this paper for activator nets.
- In such cases some kind of interval step sequence semantics might be very useful.

Interval Step Sequence Semantics of Activator Nets

Let $m_0 = \{s_1, s_2\}, m_F = \{s_3, s_4\}$ and let $\mathrm{FISS}_N(m \rightsquigarrow m')$ denote Firing Interval Step Sequences of the net N from m to m'.



$\mathrm{FSS}_{AN_2}(m_0 \rightsquigarrow m_F) = \mathrm{FSS}_{IN_2}(m_0 \rightsquigarrow m_F) = \{\{a,b\}\}$,

$\mathrm{FISS}_{AN_2}(m_0 \rightsquigarrow m_F) \stackrel{df}{=} \mathrm{FSS}_{\widehat{AN_2}}(m_0 \rightsquigarrow m) =$
$\{\{B_a, B_b\}\{E_a, E_b\}, \{B_a, B_b\}\{E_a\}\{E_b\}, \{B_a, B_b\}\{E_b\}\{E_a\}\}$.

$\mathrm{SO}_{AN_2}(m_0 \rightsquigarrow m) = \mathrm{IO}_{AN_2}(m_0 \rightsquigarrow m) = \{\blacktriangleleft_{\{B_a,B_b\}\{E_a,E_b\}}\}$.

- **The step sequence semantics of activator nets and the interval step sequence semantics of activator nets are CONSISTENT.**

Summary

- The results of this paper emphasize the difference between *interval semantics* and *interval order semantics*.
- Interval orders are partial orders so for different a and b we have either $a \prec b$ or $a \frown b$, i.e. only two possible relationships: *less than* and *not comparable*.
- For two intervals a and b, we might have up to seven relationships: *a before b, a equal b, a meets b, a overlaps b, a during b, a starts b* and *a finishes b*.
- Interval order semantics is usually simpler, but not necessarily always equivalent to interval semantics.
- When a model of concurrent system $\mathcal{M}$ is *sound*, then interval orders are a good abstractions of appropriate intervals; and interval order semantics and interval semantics might be considered as equivalent.
- When $\mathcal{M}$ is *not sound*, we have to be very careful when using interval orders as some of their interval representations may be invalid.
- When $\mathcal{M}$ is *not sound*, interval semantics in terms of interval sequences or interval step sequences may still be well defined and valid, but interval orders may not.

THANK YOU!
QUESTIONS?