

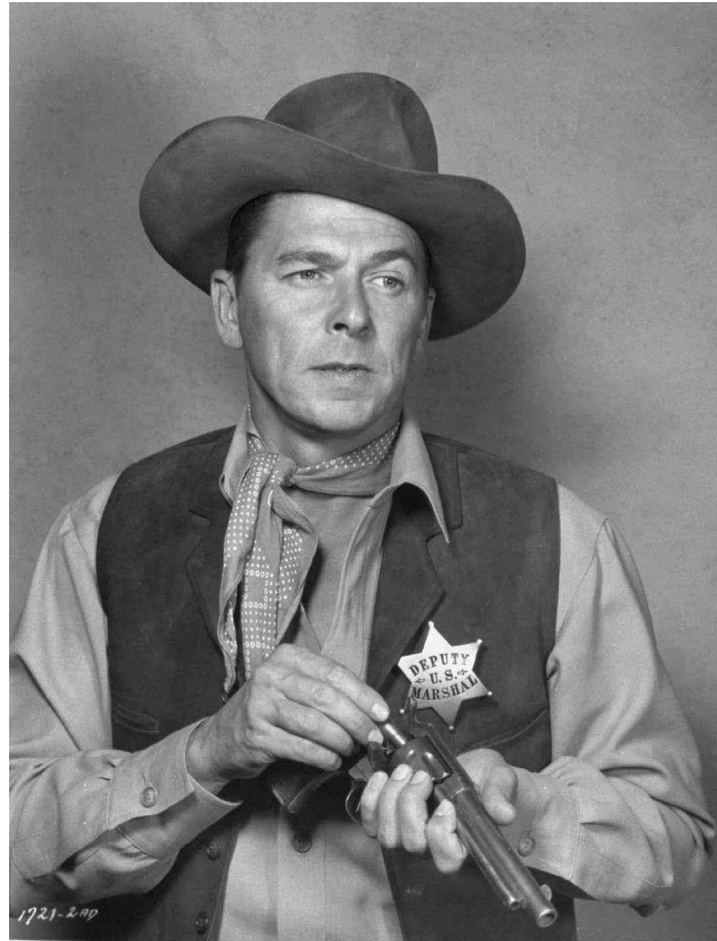
25 Years (more or less . . . )  
of  
Net Unfoldings  
and  
True-Concurrency Analysis Tools

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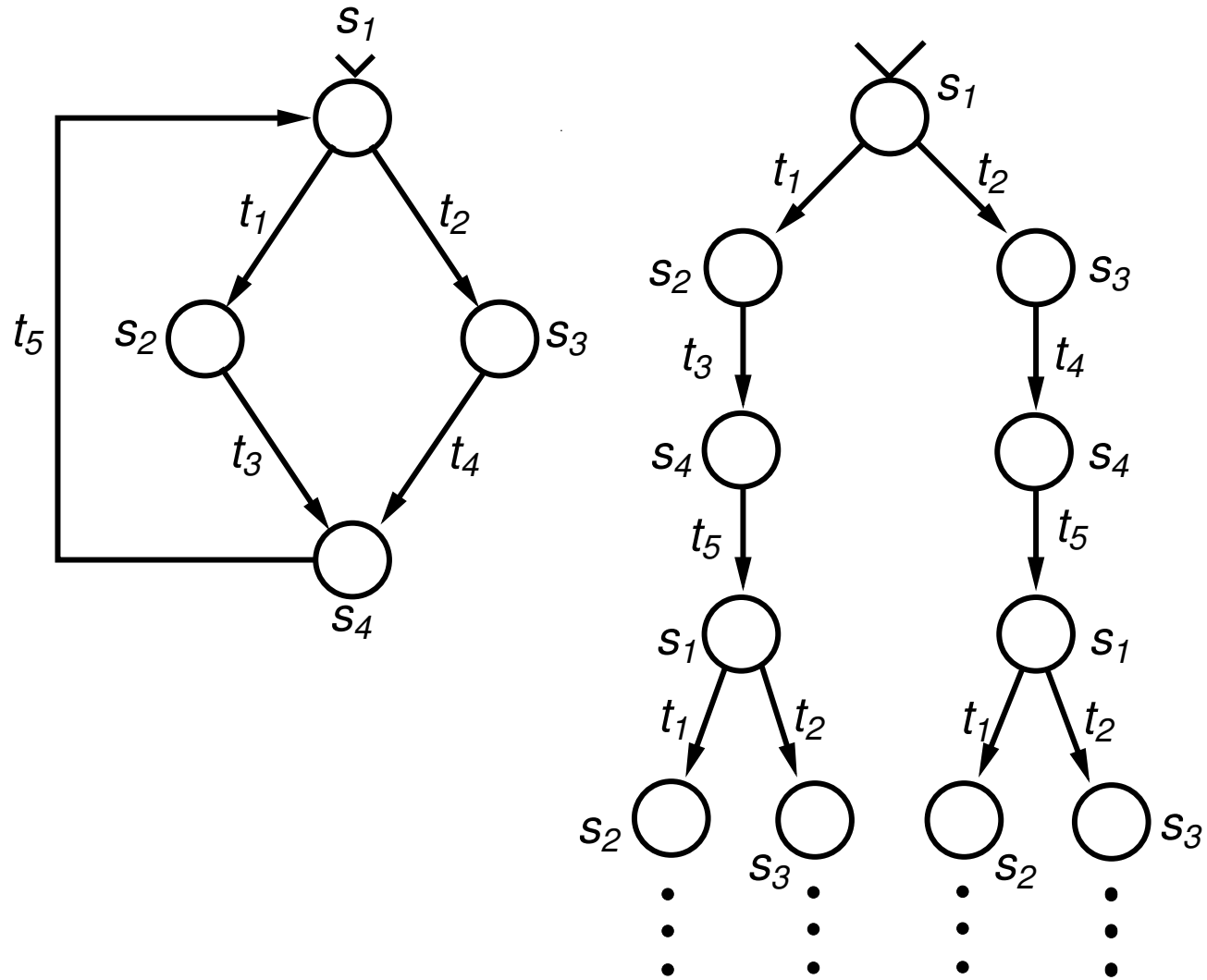
# 1981

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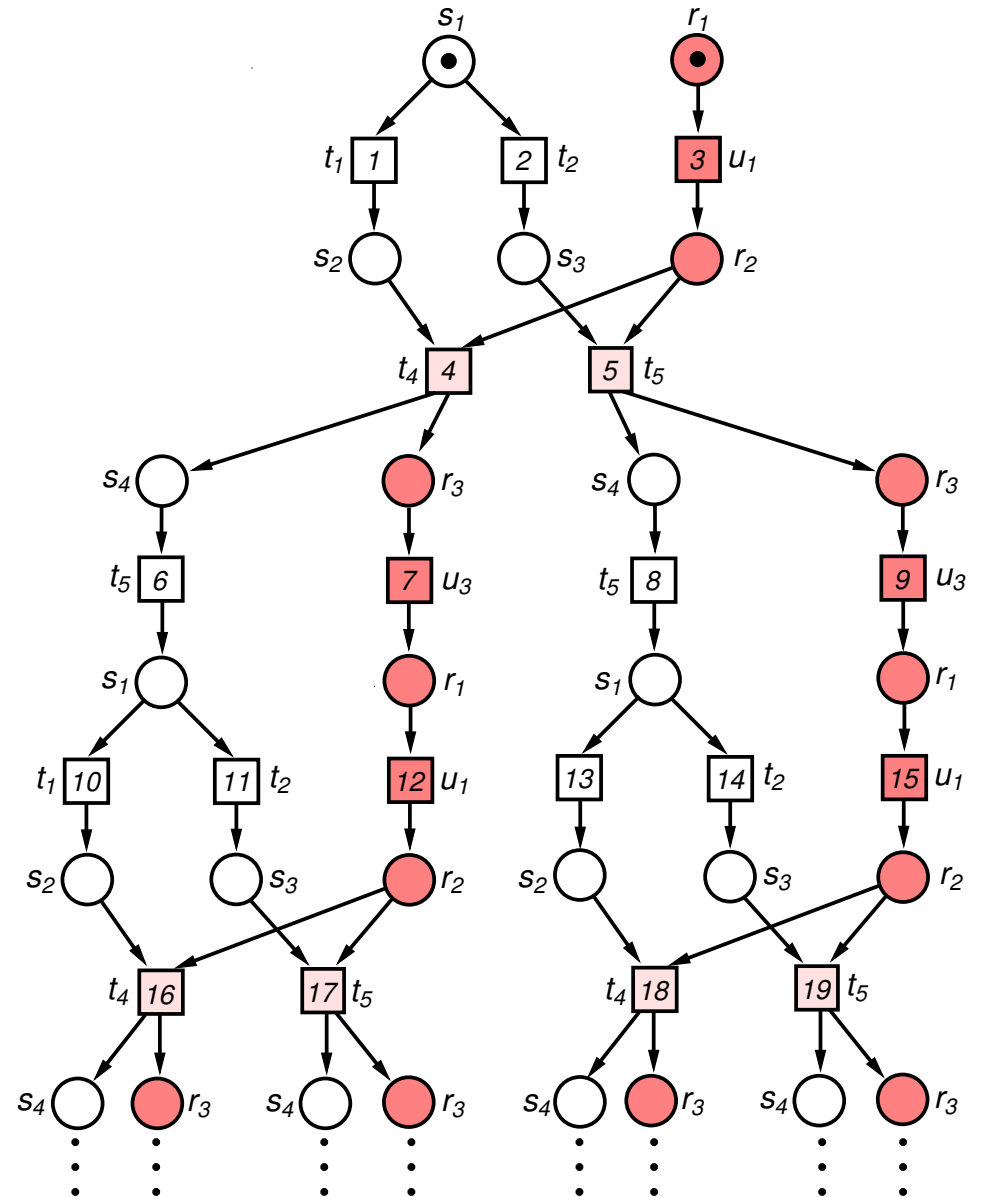
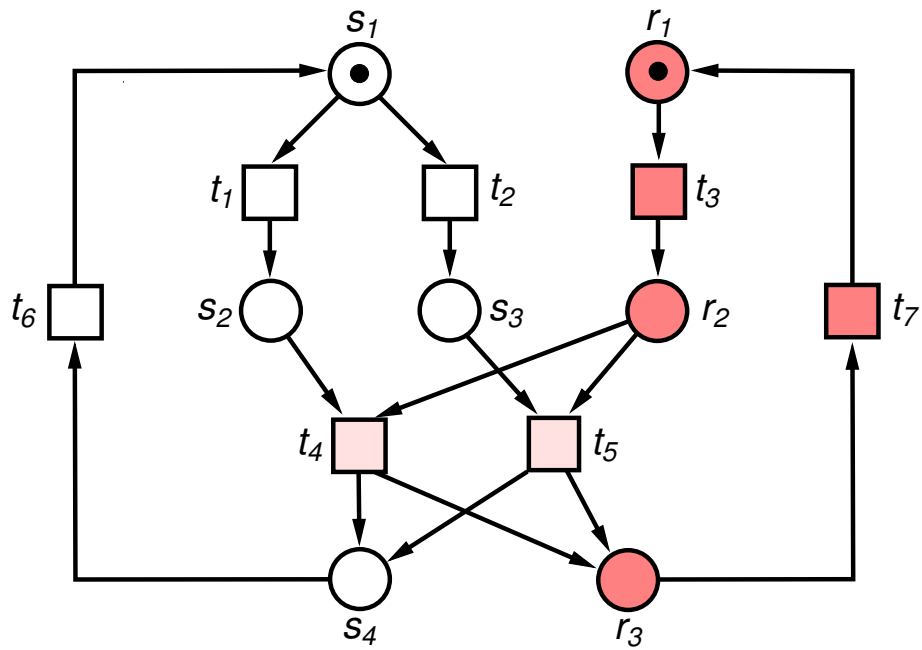


# Unfolding of a transition system

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# Nielsen, Plotkin, Winskel '81: Petri nets can also be unfolded



# Nielsen, Plotkin, Winskel '81: Petri nets can also be unfolded

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- Motivation: Denotational semantics of concurrent behaviour  
(extension of Scott's domain of computable functions to concurrent computation)
- During the 80s, theory of unfoldings further developed by
  - Winskel (synchronization trees '84, event structures '86)
  - Engelfriet (branching processes '91)

1992

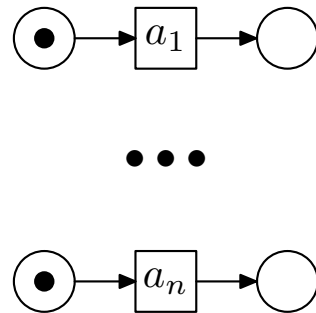
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# McMillan: Can unfoldings help to fight state-explosion ?

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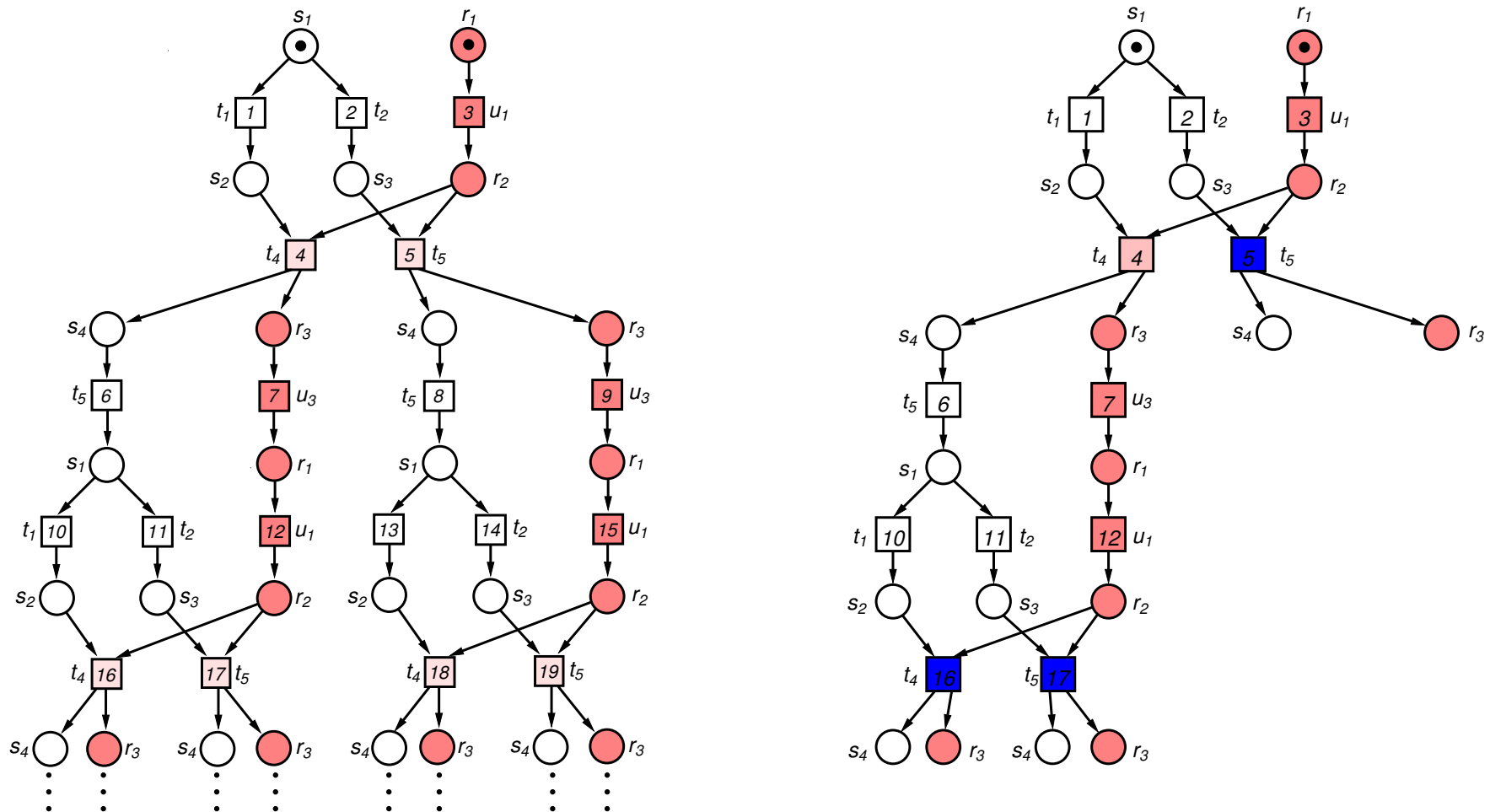
- A system composed of  $n$  independent components



- has  $2^n$  reachable states, but
- its unfolding is the system itself, and has size  $O(n)$
- **Question:** Can we base verification on the unfolding?
- **Obstacle:** the unfolding is in most cases an infinite object!

# Cut-off events and complete prefixes

- **Solution:** Construct a **complete prefix** of the unfolding containing all reachable states by identifying **cut-off events**





# Cut-off events and complete prefixes

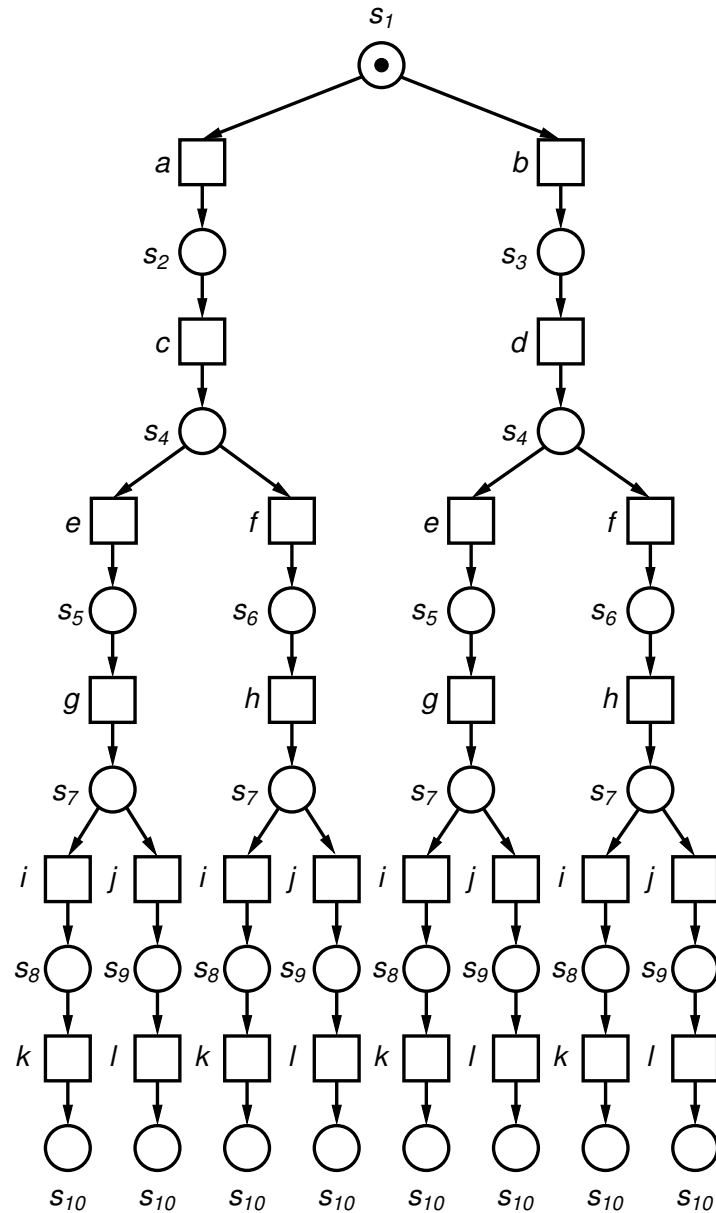
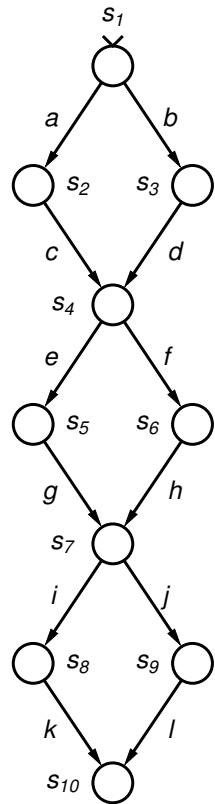
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However, in the worst case McMillan's complete prefix could be

**exponentially larger**

than the reachability graph!

# Cut-off events and complete prefixes



1996

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## E., Römer, Vogler '96: Size-guarantee

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- **Adequate orders**: orders on the events of the unfolding such that
  - if events added in this order, and
  - cut-offs identified as in McMillan's approachthen the prefix so constructed is complete.
- **Total** adequate orders guarantee that number of events **never exceeds** number of reachable markings.
- Problem of McMillan's approach: His order was partial
- ERV '96 found the first total adequate order; others followed  
(E., Römer '99; Niebert, Qu '06)

1999

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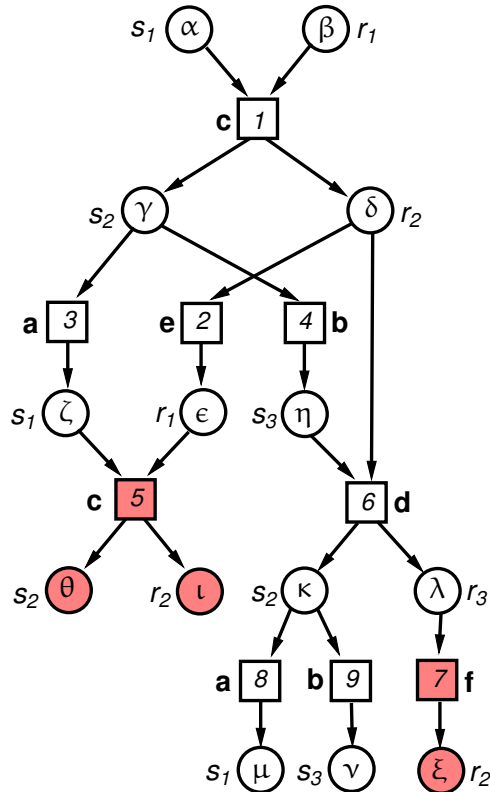


# Extracting information from complete prefixes

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- Complete prefixes are a “compact encoding” of the state space, but reachability information must be “extracted” from them.
- [Heljanko and Khomenko \(PhD theses, several papers\)](#): Reachability queries can be solved very efficiently using SAT / ILP.

# Extracting information from complete prefixes



place	clause
$\alpha$	$\alpha \leftrightarrow \neg 1$
$\beta$	$\beta \leftrightarrow \neg 1$
$\gamma$	$((3 \vee 4) \rightarrow 1) \wedge \neg(3 \wedge 4)$ $\wedge (\gamma \leftrightarrow (1 \wedge \neg 3 \wedge \neg 4))$
$\delta$	$((2 \vee 6) \rightarrow 1) \wedge \neg(2 \wedge 6)$ $\wedge (\delta \leftrightarrow (1 \wedge \neg 2 \wedge \neg 6))$
...	
$\xi$	$\xi \leftrightarrow 9$

- Further progress in SAT and SMT solving has turned the extraction problem into a non-issue.

2000

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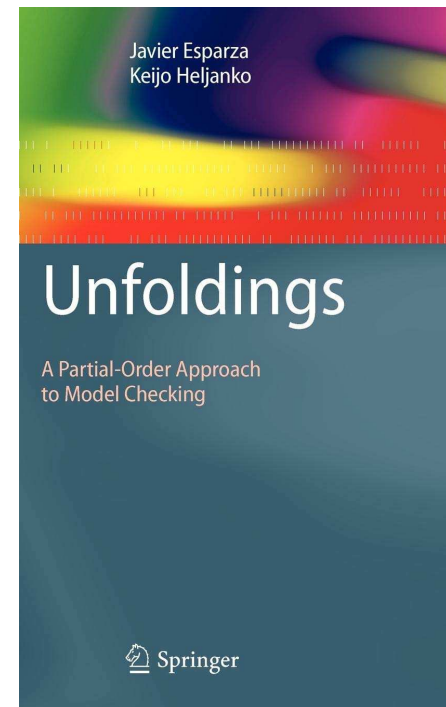


# From reachability to model-checking LTL

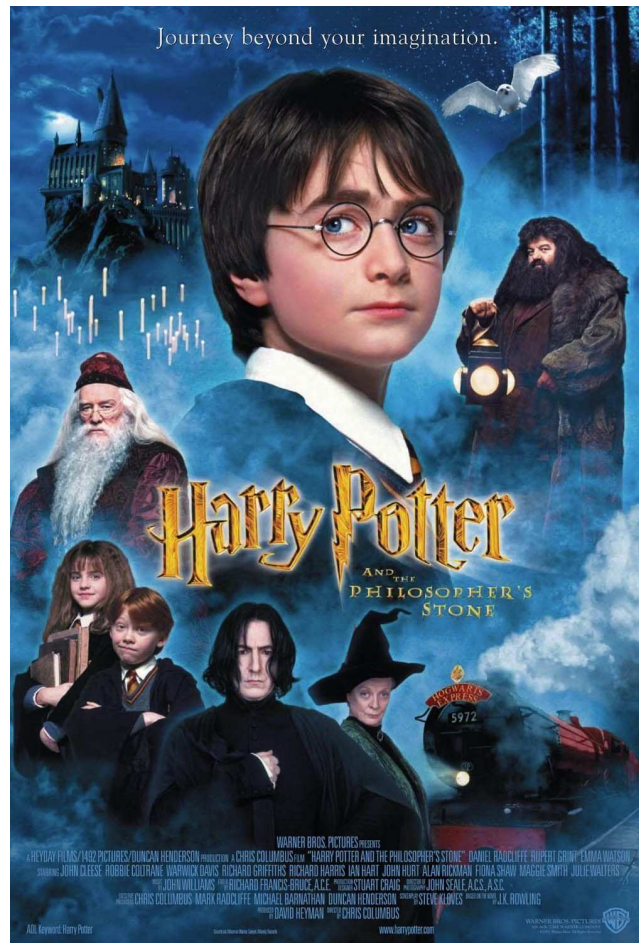
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- Two unfolding-based algorithms to model-check arbitrary (next-free) LTL properties presented at ICALP '00  
(Couvreur, Grivet, Poitrenaud; E., Heljanko)
- The algorithm by E. and Heljanko is described in

E., Heljanko:  
Unfoldings  
A Partial Order Approach  
to Model Checking  
Springer, 2008



# 2000-2010



# Theory

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- Parallel and distributed generation of the unfolding  
(Baldan, Haar, Heljanko, Khomenko, König, Koutny ...)
- Even more compact representations: Merged processes  
(Khomenko, Koutny, Rodriguez, Schwoon, Vogler ...)
- Extensions to more general models
  - Contextual nets (Baldan, Rodriguez, Schwoon, Vogler, Yakovlev ...)
  - High-level nets (Khomenko, Koutny, Schöter ...)
  - Timed models (Bouyer, Cassez, Chatain, Haddad, Jard ...)

# Tools

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- PEP (Oldenburg, Best, Stehno, ...)
- Mole (Schwoon)
- Unfolding Tools (Khomenko)
- unfsmodels, mcsmodels (Heljanko)

# Applications

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- **Analysis of asynchronous circuits**
  - Circuits specified as interpreted Petri nets
  - Concurrent Asynchronous Systems Group, University of Newcastle: tool-chain for verification and fault-fixing of STGs based on unfoldings  
(Khomenko, Koutny, Vogler, Yakovlev ...)
- **Monitoring and diagnosis**
  - Distributed systems with alarms attached to some nodes
  - Problem: find **cause** of the alarms → true-concurrency approach
  - IRISA group in Rennes, MEXICO project at ENS Cachan: diagnosis tools  
(Benveniste, Chatain, Haar, Jard, Schwoon ...)

# Applications

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- **Verification of graph transformation systems**
  - Unfolding used to overapproximate the set of reachable graphs  
(Baldan, Corradini, König, Kozioura ...)
- **AI Planning** (Bonet, Haslum, Hickmott, Khomenko, Vogler, ...)

# 2010-today

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# Applications (2010-today)

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- **Systems Biology**
  - Boolean networks used to model cellular regulatory processes
  - Unfoldings give compact representation of the reachable transitions (Pauleve, Chatain, Haar, Schwoon, ...)
- **Testing and verification of multithreaded programs**
  - Unfolding used to generate small set of test cases with high coverage (Heljanko, Kähkönen, Ponce de Leon, Saarikivi ...)
  - Unfolding used to guide partial-order reduction (Rodriguez, Sousa, Petrucci, Kröning ...)
- **Process discovery** (Carmona, Ponce de Leon, Rodriguez ...)



# Conclusions

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- Straight line from Petri's nonsequential processes to concrete algorithms, tools, and application domains
- (Most?) successful spin-off of true-concurrency semantics
- Turning point: verification through algorithmic construction of semantic objects
- True-concurrency useful in two ways:
  - Compact representation of state spaces
  - Information about causality and independence
- Blockchain ?